

Chapter 1 ~ Function

Date:

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Exercise - 1.1

Q.1. Solⁿ.

(a) Here,

$$f = \{(1, 8), (2, 9), (3, 10)\}$$

$$g = \{(8, 5), (9, 6), (10, 7)\}$$

Now,

$$g \circ f = \{(1, 5), (2, 6), (3, 7)\}$$

(b) Here,

$$f = \{(1, 2), (3, 6), (5, 10)\}$$

$$g = \{(2, 4), (6, 12), (10, 20)\}$$

Now,

$$g \circ f = \{(1, 4), (3, 12), (5, 20)\}$$

Q.2. Solⁿ.

(a) Here,

$$f(x) = 2x$$

$$g(x) = x^2$$

Now,

$$f \circ g = f(g(x))$$

$$= f(x^2)$$

$$= 2 \times x^2$$

$$= 2x^2$$

Also,

$$\begin{aligned}g \circ f &= g(f(x)) \\&= g(2x) \\&= (2x)^2 \\&= 4x^2\end{aligned}$$

(b) Here,

$$f(x) = 2x + 3$$

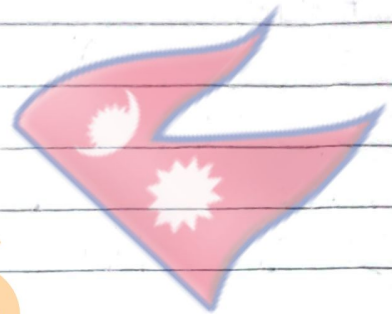
$$g(x) = 3x^2$$

Now,

$$\begin{aligned}f \circ g &= f(g(x)) \\&= f(3x^2) \\&= 2 \times 3x^2 + 3 \\&= 6x^2 + 3\end{aligned}$$

Also,

$$\begin{aligned}g \circ f &= g(f(x)) \\&= g(2x + 3) \\&= 3 \times (2x + 3)^2 \\&= 3 \times (4x^2 + 12x + 9) \\&= 12x^2 + 36x + 27\end{aligned}$$



(c) Here,

$$f(x) = x^2 + 2$$

$$g(x) = 2x + 1$$

Now,

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= f(2x+1) \\ &= (2x+1)^2 + 2 \\ &= 4x^2 + 4x + 1 + 2 \\ &= 4x^2 + 4x + 3 \end{aligned}$$

Also,

$$\begin{aligned} g \circ f &= g(f(x)) \\ &= g(x^2 + 2) \\ &= 2x(x^2 + 2) + 1 \\ &= 2x^3 + 4x + 1 \\ &= 2x^3 + 4x + 1 \end{aligned}$$

(d) Here,

$$f(x) = 2x^2 + 1$$

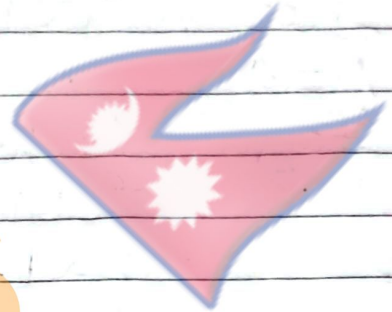
$$g(x) = 4x + 5$$

Now,

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= f(4x+5) \\ &= 2(4x+5)^2 + 1 \\ &= 2(16x^2 + 40x + 25) + 1 \\ &= 32x^2 + 80x + 50 + 1 \\ &= 32x^2 + 80x + 51 \end{aligned}$$

Also,

$$\begin{aligned}
 g \circ f &= g(f(x)) \\
 &= g(2x^2+1) \\
 &= 4 \times (2x^2+1) + 5 \\
 &= 8x^2 + 4 + 5 \\
 &= 8x^2 + 9
 \end{aligned}$$



(e) Here,

$$f(x) = 2x^2 - 3$$

$$g(x) = 3x - 2$$

Now,

$$\begin{aligned}
 f \circ g &= f(g(x)) \\
 &= f(3x-2) \\
 &= 2 \times (3x-2)^2 - 3 \\
 &= 2(9x^2 - 12x + 4) - 3 \\
 &= 18x^2 - 24x + 8 - 3 \\
 &= 18x^2 - 24x + 5
 \end{aligned}$$

Also,

$$\begin{aligned}
 g \circ f &= g(f(x)) \\
 &= g(2x^2-3) \\
 &= 3 \times (2x^2-3) - 2 \\
 &= 6x^2 - 9 - 2 \\
 &= 6x^2 - 11
 \end{aligned}$$

(f) Here,

$$f(x) = 4x + 5$$

$$g(x) = 8x - 7$$

Now,

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= f(8x - 7) \\ &= 4 \times (8x - 7) + 5 \\ &= 32x - 28 + 5 \\ &= 32x - 23 \end{aligned}$$

Also,

$$\begin{aligned} g \circ f &= g(f(x)) \\ &= g(4x + 5) \\ &= 8 \times (4x + 5) - 7 \\ &= 32x + 40 - 7 \\ &= 32x + 33 \end{aligned}$$

Q. 3. Solⁿ

(a) Here,

$$f(x) = x^2 + 5$$

$$g(x) = x^3 + 6$$

Now,

$$\begin{aligned} f \circ g &= f(g(x)) \\ &= f(x^3 + 6) \\ &= (x^3 + 6) + 5 \\ &= x^3 + 11 \end{aligned}$$

$$f \circ g(3) =$$

Q.3 Solⁿ

(a) Here,

$$f(x) = x^2 + 5$$

$$g(x) = x^3 + 6$$

Now,

$$g(3) = 3^3 + 6 = 27 + 6 = 33$$

$$f(2) = 2^2 + 5 = 4 + 5 = 9$$

$$\begin{aligned}\therefore fog(3) &= f(g(3)) \\ &= f(33) \\ &= 33^2 + 5 \\ &= 1089 + 5 \\ &= 1094\end{aligned}$$

Also,

$$\begin{aligned}gof(2) &= g(f(2)) \\ &= g(9) \\ &= 9^3 + 6 \\ &= 729 + 6 \\ &= 735\end{aligned}$$

(b) Here,

$$f(x) = x^4 + 4x$$

$$g(x) = x^2 + 5$$

Now,

$$f(2) = 2^4 + 4 \times 2 = 16 + 8 = 24$$

$$g(3) = 3^2 + 5 = 9 + 5 = 14$$

$$\begin{aligned}
 \therefore f \circ g(3) &= f(g(3)) \\
 &= f(14) \\
 &= 14^4 + 4 \times 14 \\
 &= 38416 + 56 \\
 &= 38472
 \end{aligned}$$

Also,

$$\begin{aligned}
 g \circ f(2) &= g(f(2)) \\
 &= g(24) \\
 &= 24^2 + 5 \\
 &= 576 + 5 \\
 &= 581
 \end{aligned}$$

(c) Here,

$$f(x) = x^3 + 3x$$

$$g(x) = x^2 + 2x$$

Now,

$$f(2) = 2^3 + 3 \times 2 = 8 + 6 = 14$$

$$g(3) = 3^2 + 2 \times 3 = 9 + 6 = 15$$

$$\begin{aligned}
 \therefore f \circ g(3) &= f(15) \\
 &= 15^3 + 3 \times 15 \\
 &= 3375 + 45 \\
 &= 3420
 \end{aligned}$$

Also,

$$\begin{aligned}
 g \circ f(2) &= g(14) \\
 &= 14^2 + 2 \times 14 \\
 &= 196 + 28 \\
 &= 224
 \end{aligned}$$

(d) Here,

$$f(x) = 2x^2 - 3x$$

$$g(x) = 4x - 8$$

Now,

$$f(2) = 2 \times 2^2 - 3 \times 2 = 2 \times 4 - 6 = 2$$

$$g(3) = 4 \times 3 - 8 = 12 - 8 = 4$$

$$\therefore f \circ g(3) = f(g(3))$$

$$= f(4)$$

$$= 2 \times 4^2 - 3 \times 4$$

$$= 2 \times 16 - 12$$

$$= 32 - 12$$

$$= 20$$

Also,

$$g \circ f(2) = g(f(2))$$

$$= g(2)$$

$$= 4 \times 2 - 8$$

$$= 8 - 8$$

$$= 0$$

(e) Here,

$$f(x) = x^2 + 2x + 3$$

$$g(x) = x^2 + 4x + 5$$

Now,

$$f(2) = 2^2 + 2 \times 2 + 3 = 4 + 4 + 3 = 11$$

$$g(3) = 3^2 + 4 \times 3 + 5 = 9 + 12 + 5 = 26$$

$$\begin{aligned}\therefore f \circ g(3) &= f(g(3)) \\ &= f(26) \\ &= 26^2 + 2 \times 26 + 3 \\ &= 676 + 52 + 3 \\ &= 731\end{aligned}$$

Also,

$$\begin{aligned}g \circ f(2) &= g(f(2)) \\ &= g(11) \\ &= 11^2 + 4 \times 11 + 5 \\ &= 121 + 44 + 5 \\ &= 170\end{aligned}$$

(f) Here,

$$f(x) = x^2 - 2x + 8$$

$$g(x) = x^2 - 5x + 6$$

Now,

$$f(2) = 2^2 - 2 \times 2 + 8 = 4 - 4 + 8 = 8$$

$$g(3) = 3^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$$

$$\begin{aligned}\therefore f \circ g(3) &= f(g(3)) \\ &= f(0) \\ &= 0^2 - 2 \times 0 + 8 \\ &= 0 - 0 + 8 \\ &= 8\end{aligned}$$

Also,

$$\begin{aligned}g \circ f(2) &= g(f(2)) \\ &= g(8)\end{aligned}$$

$$\begin{aligned}
 &= 8^2 - 5 \times 8 + 6 \\
 &= 64 - 40 + 6 \\
 &= 30
 \end{aligned}$$

Q.4 Solⁿ

(a) Here,

$$f(x) = 3x + b$$

$$ff(2) = 12$$

Now,

$$f(2) = 3 \times 2 + b = 6 + b$$

$$\therefore ff(2) = f(6 + b)$$

$$\text{or, } 12 = 3 \times (6 + b) + b$$

$$\text{or, } 12 = 18 + 3b + b$$

$$\text{or, } 12 - 18 = 4b$$

$$\text{or, } -6 = 4b$$

$$\text{or, } b = -\frac{6}{4}$$

$$\therefore b = -\frac{3}{2}$$

(b) Here,

$$f(x) = 3x$$

$$g(x) = x + 2$$

$$fog(x) = 18$$

Now,

$$f \circ g(x) = f(g(x))$$

$$\text{or, } 18 = f(x+2)$$

$$\text{or, } 18 = 3x(x+2)$$

$$\text{or, } 18 = 3x+6$$

$$\text{or, } 18-6 = 3x$$

$$\text{or, } 3x = 12$$

$$\therefore x = 4$$

Q.5 Soln

(a) Here,

$$f(x) = 4x$$

$$g(x) = x+1$$

$$f \circ g(x) = 20$$

Now,

$$f \circ g(x) = f(g(x))$$

$$\text{or, } 20 = f(x+1)$$

$$\text{or, } 20 = 4x(x+1)$$

$$\text{or, } 20 = 4x+4$$

$$\text{or, } 20-4 = 4x$$

$$\text{or, } 16 = 4x$$

$$\therefore x = 4$$

(b) Here,

$$f(x) = 2x$$

$$g(x) = x + 2$$

$$fg(x) = 8$$

Now,

$$fg(x) = f(g(x))$$

$$\text{or, } 8 = f(x+2)$$

$$\text{or, } 8 = 2x(x+2)$$

$$\text{or, } 8 = 2x + 4$$

$$\text{or, } 8 - 4 = 2x$$

$$\text{or, } 4 = 2x$$

$$\therefore x = 2$$

6(a) Here,

$$f(x) = x + 2 \Rightarrow f(4) = 4 + 2 = 6$$

$$g(x+1) = x+1$$

$$\text{Let, } x+1 = a$$

$$\therefore x = a - 1$$

Then,

$$g(a-1+1) = a-1+1$$

$$\text{or, } g(a) = a$$

$$\therefore g(x) = x$$

Now,

$$\begin{aligned} g \circ f(4) &= g(6) \\ &= 6 \end{aligned}$$

(b) Here,

$$f(x) = x+2 \Rightarrow f(8) = 8+2 = 10$$

$$g(x+2) = x+2$$

$$\text{Let, } x+2 = a$$

$$\therefore x = a-2$$

$$g(a-2+2) = a-2+2$$

$$\text{or, } g(a) = a$$

$$\therefore g(x) = x$$

Now,

$$\begin{aligned} g \circ f(8) &= g(f(8)) \\ &= g(10) \\ &= 10 \end{aligned}$$



Q.7 Solⁿ

(a) Here,

$$f(4x-15) = 8x-27$$

$$\text{Let } 4x-15 = z$$

$$\text{or, } 4x = z+15$$

$$\therefore x = \frac{z+15}{4}$$

Then,

$$f\left(4 \times \frac{z+15}{4} - 15\right) = 8 \times \frac{z+15}{4} - 27$$

$$\text{or, } f(z) = 2z + 30 - 27$$

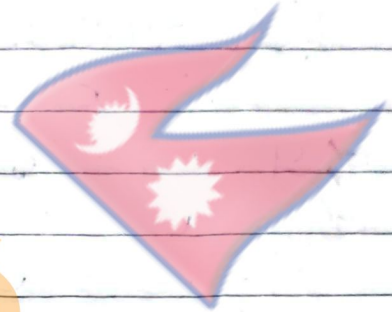
$$\text{or, } f(z) = 2z + 3$$

$$\therefore f(x) = 2x + 3$$

Now,

$$f(2) = 2 \times 2 + 3 = 7$$

$$\begin{aligned} \therefore f \circ f(2) &= f(f(2)) \\ &= f(7) \\ &= 2 \times 7 + 3 \\ &= 14 + 3 \\ &= 17 \end{aligned}$$



(b) Here,

$$g(2x-5) = 8x-19$$

$$\text{Let, } 2x-5 = z$$

$$\text{or, } 2x = z+5$$

$$\therefore x = \frac{z+5}{2}$$

Then,

$$g\left(2 \times \frac{z+5}{2} - 5\right) = 8 \times \frac{z+5}{2} - 19$$

$$\text{or, } g(z) = 4z + 20 - 19$$

$$\therefore g(x) = 4x + 1$$

$$\Rightarrow g(2) = 4 \times 2 + 1 = 9$$

Now,

$$\begin{aligned} g \circ g(2) &= g(9) \\ &= 4 \times 9 + 1 \\ &= 36 + 1 \\ &= 37 \end{aligned}$$

8. Solⁿ

(a) Here,

$$f(3x-1) = -6x+3$$

Let, $3x-1 = z$

$$\text{or, } 3x = z+1$$

$$\text{or, } x = \frac{z+1}{3}$$

Then,

$$f\left(3 \times \frac{z+1}{3} - 1\right) = -6^2 \times \left(\frac{z+1}{3}\right) + 3$$

$$\text{or, } f(z) = -2z - 2 + 3$$

$$\text{or, } f(z) = -2z + 1$$

$$\therefore f(x) = 1 - 2x$$

$$\Rightarrow f(2) = 1 - 2 \times 2 = 1 - 4 = -3$$

Now,

$$\begin{aligned} f \circ f(2) &= f(f(2)) \\ &= f(-3) \\ &= 1 - 2 \times (-3) \\ &= 7 \end{aligned}$$

(b) Here,

$$f(2x+5) = 6x + 19$$

$$\text{Let, } 2x+5 = z$$

$$\text{or, } 2x = z - 5$$

$$\therefore x = \frac{z-5}{2}$$

Then,

$$f\left(2 \times \frac{z-5}{2} + 5\right) = 6 \times \left(\frac{z-5}{2}\right) + 19$$

$$\text{or, } f(z) = 3z - 15 + 19$$

$$\therefore f(x) = 3x + 4$$

$$\Rightarrow f(2) = 3 \times 2 + 4 = 10$$

Now,

$$\begin{aligned} f \circ f(2) &= f(f(2)) \\ &= f(10) \\ &= 3 \times 10 + 4 \\ &= 30 + 4 \\ &= 34 \end{aligned}$$

Q.9. (a) Soln

Given,

$$f(x) = x + 2$$

$$g(x) = x^2$$

$$k(x) = 3x + 1$$

(i) Here,

$$k(2) = 3 \times 2 + 1 = 7$$

$$\therefore f \circ k(2) = f(k(2))$$

$$= f(7)$$

$$= 7 + 2$$

$$= 9$$

(ii) Here,

$$k(2) = 3 \times 2 + 1 = 7$$

$$\therefore k \circ k(2) = k(k(2))$$

$$= k(7)$$

$$= 3 \times 7 + 1$$

$$= 21 + 1$$

$$= 22$$

(iii) Here,

$$k \circ k(x) = k(k(x))$$

$$= k(3x + 1)$$

$$= 3 \times (3x + 1) + 1$$

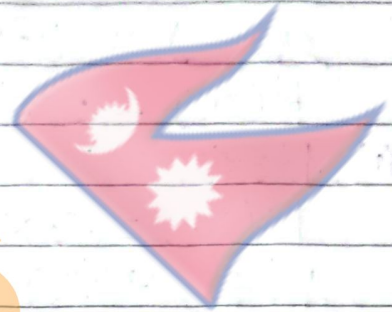
$$= 9x + 3 + 1$$

$$= 9x + 4$$

$$\begin{aligned}
 (k \circ k)(-5) &= g \times (-5) + 4 \\
 &= -45 + 4 \\
 &= -41
 \end{aligned}$$

Now,

$$\begin{aligned}
 f(k \circ k)(-5) &= f(-41) \\
 &= -41 + 2 \\
 &= -39
 \end{aligned}$$



(iv) Here,

$$\begin{aligned}
 g \circ k(x) &= g(k(x)) \\
 &= g(3x+1) \\
 &= (3x+1)^2 \\
 &= (3x)^2 + 2 \times 3x \times 1 + 1^2 \\
 &= 9x^2 + 6x + 1
 \end{aligned}$$

Now,

$$\begin{aligned}
 f(g \circ k)(x) &= f(9x^2 + 6x + 1) \\
 &= 9x^2 + 6x + 1 + 2 \\
 &= 9x^2 + 6x + 3
 \end{aligned}$$

(b) Solⁿ

Given,

$$f(x) = 3x - 4$$

$$h(x) = x + 3$$

$$g(x) = -2x + 1$$

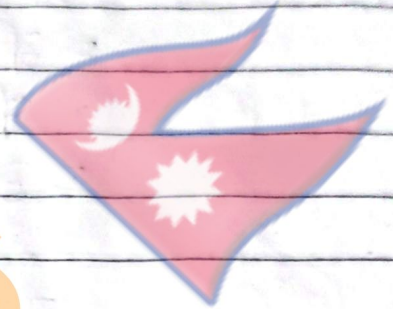
(i) Here,

$$\begin{aligned}
 f \circ h &= f(h(x)) \\
 &= f(x+3)
 \end{aligned}$$

$$\begin{aligned} &= 3x(x+3) - 4 \\ &= 3x^2 + 9 - 4 \\ &= 3x^2 + 5 \end{aligned}$$

(ii) Here,

$$\begin{aligned} h \circ h &= h \circ h(x) \\ &= h(x+3) \\ &= x+3+3 \\ &= x+6 \end{aligned}$$



(iii) $g \circ h(f) = (g \circ h) f(x)$

$$\begin{aligned} \Rightarrow g \circ h &= g(h(x)) \\ &= g(x+3) \\ &= -2(x+3) + 1 \\ &= -2x - 6 + 1 \\ &= -2x - 5 \end{aligned}$$

Now

$$\begin{aligned} (g \circ h) f(x) &= (g \circ h)(3x-4) \\ &= -2(3x-4) - 5 \\ &= -6x + 8 - 5 \\ &= -6x + 3 \end{aligned}$$

(iv) Here,

$$\begin{aligned}
 f(goh) &= f(g(h(x))) \\
 &= f(g(x+3)) \\
 &= f(-2x(x+3)+1) \\
 &= f(-2x-6+1) \\
 &= f(-2x-5) \\
 &= 3x(-2x-5)-4 \\
 &= -6x-15-4 \\
 &= -6x-19
 \end{aligned}$$

(c) Here,

$$f(x) = 2x-1$$

$$g(x) = 3x$$

$$h(x) = x+3$$

$$(f \circ g \circ h)(x) = -f(x)$$

Now,

$$(f \circ g \circ h)(x) = f \circ g \circ h(x)$$

$$\text{or, } -(2x-1) = f \circ g(x+3)$$

$$\text{or, } -(2x-1) = f(3x(x+3))$$

$$\text{or, } -2x+1 = f(3x+9)$$

$$\text{or, } -2x+1 = 2x(3x+9)-1$$

$$\text{or, } -2x+1 = 6x+18-1$$

$$\text{or, } -2x-6x = 17-1$$

$$\text{or, } -8x = 16$$

$$\therefore x = -2$$

$$\begin{aligned}
 \text{Again, } g \circ f \circ h &= g \circ f(x+3) \\
 &= g(2 \times (x+3) - 1) \\
 &= g(2x+6-1) \\
 &= g(2x+5) \\
 &= 3 \times (2x+5) \\
 &= 6x+15
 \end{aligned}$$

$$\therefore (g \circ f \circ h)(x) = 6x+15$$

Now,

$$\begin{aligned}
 (g \circ f \circ h)(2) &= 6 \times 2 + 15 \\
 &= 12 + 15 \\
 &= 27
 \end{aligned}$$

Q.N. 10. Solⁿ

(a) Here,

$$f(x) = 2x+3 \Rightarrow f(2) = 2 \times 2 + 3 = 7$$

$$g \circ f(x) = 2x-1$$

Now,

$$g \circ f(x) = g(f(x))$$

$$\text{or, } 2x-1 = g(2x+3)$$

$$\text{or, } g(2x+3) = 2x-1$$

$$\text{Let, } 2x+3 = a$$

$$\text{or, } 2x = a-3$$

$$\therefore x = \frac{a-3}{2}$$

Then,

$$g\left(2 \times \frac{a-3}{2} + 3\right) = 2 \times \frac{a-3}{2} - 1$$

$$\text{or, } g(a) = a - 3 - 1$$

$$\therefore g(x) = x - 4$$

$$\Rightarrow g(2) = 2 - 4 = -2$$

Now,

$$\begin{aligned} f \circ g(2) &= f(g(2)) \\ &= f(-2) \\ &= 2 \times (-2) + 3 \\ &= -4 + 3 \\ &= -1 \end{aligned}$$

Also,

$$\begin{aligned} g \circ f(2) &= g(f(2)) \\ &= g(7) \\ &= 7 - 4 \\ &= 3 \end{aligned}$$

(b) Here,

$$f(x) = x^2 \Rightarrow f(2) = 2^2 = 4$$

$$g \circ f(x) = x^2 - 3$$

Now,

$$g \circ f(x) = g(f(x))$$

$$\text{or, } x^2 - 3 = g(x^2)$$

$$\text{or, } g(x^2) = x^2 - 3$$

$$\text{Let } x^2 = a$$

$$\therefore x = \sqrt{a}$$

Then,

$$g((\sqrt{a})^2) = (\sqrt{a})^2 - 3$$

$$\text{or, } g(a) = a - 3$$

$$\therefore g(x) = x - 3$$

$$\Rightarrow g(2) = 2 - 3 = -1$$

Now,

$$f \circ g(2) = f(g(2))$$

$$= f(-1)$$

$$= (-1)^2$$

$$= 1$$

Also,

$$g \circ f(2) = g(f(2))$$

$$= g(4)$$

$$= 4 - 3$$

$$= 1$$

Q.11 80/11

(a) Here,

$$f(x+1) = x+3$$

$$\text{Let, } x+1 = z$$

$$\therefore x = z-1$$

Then,

$$f(z-1+1) = z-1+3$$

$$\text{or, } f(z) = z+2$$

$$\therefore f(x) = x+2$$

Also,

$$g(x+1) = x+1$$

$$\text{Let, } x+1 = a$$

$$\therefore x = a-1$$

Then,

$$g(a-1+1) = a-1+1$$

$$\text{or, } g(a) = a$$

$$\therefore g(x) = x$$

$$h(x) = x^2$$

Now,

$$\text{hog} = h(g(x))$$

$$= h(x)$$

$$= x^2$$

$$\therefore (\text{hog}) \circ f = (\text{hog})(x+2)$$

$$= (x+2)^2$$

$$= x^2 + 4x + 4$$

(b) Here,

$$f(x+1) = 2x+5$$

$$\text{Let, } x+1 = a$$

$$\therefore x = a-1$$

$$\Rightarrow f(a-1+1) = 2x(a-1)+5$$

$$\text{or, } f(a) = 2a-2+5$$

$$\therefore f(x) = 2x+3$$

Also,

$$g(x+1) = 4x+5$$

$$\text{Let, } x+1 = z$$

$$\therefore x = z-1$$

$$\Rightarrow g(z-1+1) = 4x(z-1)+5$$

$$\text{or, } g(z) = 4z-4+5$$

$$\text{or, } g(z) = 4z+1$$

$$\therefore g(x) = 4x+1$$

$$h(x) = x-4$$

Now,

$$(f \circ g \circ h)x = f \circ g(h(x))$$

$$= f \circ g(x-4)$$

$$= f(4(x-4)+1)$$

$$= f(4x-16+1)$$

$$= f(4x-15)$$

$$= 2x(4x-15)+3$$

$$= 8x-30+3$$

$$= 8x-27$$

Q.12, Solⁿ.

(a) Here,

$$f(x) = x^2 + x + 2 \Rightarrow f(2) = 2^2 + 2 + 2 = 8$$

$$g(x) = 2x^2 - 7 \Rightarrow g(2) = 2 \times 2^2 - 7 = 1$$

Now,

L.H.S.

$$\begin{aligned} 2 \cdot f(g(2)) &= 2 \cdot f(1) \\ &= 2 \cdot f(1) \\ &= 2 \cdot (1^2 + 1 + 2) \\ &= 2 \cdot (1 + 1 + 2) \\ &= 2 \times 4 \\ &= 8 \end{aligned}$$

Also,

R.H.S.

$$\begin{aligned} f(2) \cdot g(2) &= 8 \times 1 \\ &= 8 \end{aligned}$$

Hence, L.H.S. = R.H.S. proved.

(b) Here,

$$f(x) = 3x^2 + 5x \Rightarrow f(4) = 3 \times 4^2 + 5 \times 4 = 68$$

$$g(x) = 4x^2 - 5 \Rightarrow g(4) = 4 \times 4^2 - 5 = 59$$

Now,

$$\begin{aligned} \text{L.H.S.} &= f(g(4)) \\ &= f(59) \\ &= 3 \times 59^2 + 5 \times 59 \\ &= 3 \times 3481 + 295 \\ &= 10738 \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= 3 f(4) \cdot g(4) - 1298 \\ &= 3 \times 68 \times 59 - 1298 \\ &= 12036 - 1298 \\ &= 10738 \end{aligned}$$

Hence, L.H.S. = R.H.S. proved.

Q. 13 Solⁿ.

(a) Here,

$$f(x) = 3x - 2$$

$$f \circ g(x) = 6x - 2$$

$$g \circ f(x) = 8$$

Now,

$$f \circ g(x) = f(g(x))$$

$$\text{or, } 6x - 2 = 3 \times g(x) - 2$$

$$\text{or, } 6x - 2 + 2 = 3 \times g(x)$$

$$\text{or, } g(x) = \frac{6x}{3}$$

$$\therefore g(x) = 2x$$

Again,

$$g \circ f(x) = g(f(x))$$

$$\text{or, } 8 = g(3x - 2)$$

$$\text{or, } 8 = 2 \times (3x - 2)$$

$$\text{or, } 4 = 3x - 2$$

$$\text{or, } 4 + 2 = 3x$$

$$\text{or, } 6 = 3x$$

$$\therefore x = 2$$

(b) Here,

$$f(x) = 4x + 5$$

$$f \circ g(x) = 8x + 13$$

$$g \circ f(x) = 28$$

Now,

$$f \circ g(x) = 8x + 13$$

$$\text{or, } f(g(x)) = 8x + 13$$

$$\text{or, } 4g(x) + 5 = 8x + 13$$

$$\text{or, } 4g(x) = 8x + 13 - 5$$

$$\text{or, } 4g(x) = 8x + 8$$

$$\text{or, } g(x) = \frac{8x + 8}{4}$$

$$\therefore g(x) = 2x + 2$$

Again,

$$g \circ f(x) = g(f(x))$$

$$\text{or, } 28 = g(4x + 5)$$

$$\text{or, } 28 = 2 \times (4x + 5) + 2$$

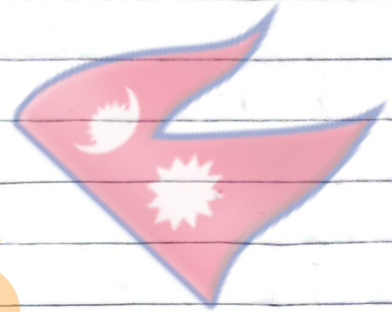
$$\text{or, } 28 = 8x + 10 + 2$$

$$\text{or, } 28 = 8x + 12$$

$$\text{or, } 28 - 12 = 8x$$

$$\text{or, } 8x = 16$$

$$\therefore x = 2$$



Q. 14 Solⁿ

(a) Here,

$$g(x) = 2x$$

$$f \circ g(x) = 6x - 2$$

$$g \circ f(x) = 10$$

Now,

$$f \circ g(x) = f(g(x))$$

$$\text{or, } 6x - 2 = f(2x)$$

$$\text{or, } f(2x) = 6x - 2$$

$$\text{Let, } 2x = a$$

$$\therefore x = \frac{a}{2}$$

Then,

$$f\left(2 \times \frac{a}{2}\right) = 6 \times \frac{a}{2} - 2$$

$$\text{or, } f(a) = 3a - 2$$

$$\therefore f(x) = 3x - 2$$

Again,

$$g \circ f(x) = g(f(x))$$

$$\text{or, } 10 = g(3x - 2)$$

$$\text{or, } 10 = 2 \times (3x - 2)$$

$$\text{or, } 5 = 3x - 2$$

$$\text{or, } 5 + 2 = 3x$$

$$\text{or, } 7 = 3x$$

$$\therefore x = \frac{7}{3}$$

(b) Here,

$$f(x) = 2x + 2$$

$$f \circ g(x) = 8x + 13$$

$$g \circ f(x) = 20$$

Now,

$$f \circ g(x) = f(g(x))$$

$$\text{or, } 8x + 13 = 2 \times g(x) + 2$$

$$\text{or, } 8x + 13 - 2 = 2 \times g(x)$$

$$\text{or, } 8x + 11 = 2 \times g(x)$$

$$\therefore g(x) = \frac{8x + 11}{2}$$

Again,

$$g \circ f(x) = g(f(x))$$

$$\text{or, } 20 = g(2x + 2)$$

$$\text{or, } 20 = \frac{8 \times (2x + 2) + 11}{2}$$

$$\text{or, } 40 = 16x + 16 + 11$$

$$\text{or, } 40 - 27 = 16x$$

$$\text{or, } 13 = 16x$$

$$\therefore x = \frac{13}{16}$$

Q.15 Soln

(a) Here,

$$f(x) = \frac{6}{x-2} \text{ where } x \neq 2$$

$$g(x) = kx^2 - 1$$

$$g \circ f(5) = 7$$

Now,

$$f(5) = \frac{6}{5-2} = \frac{6}{3} = 2$$

$$\therefore g \circ f(5) = g(2)$$

$$\therefore 7 = k \times 2^2 - 1$$

$$\therefore 7 + 1 = 4k$$

$$\therefore 8 = 4k$$

$$\therefore k = 2$$

(b) Here,

$$f(x) = \frac{a}{3-x} \text{ where } x \neq 3$$

$$g(x) = 11 - bx^2$$

$$f^2(5) = \frac{4}{5}$$

$$f \circ g(2) = \frac{1}{2}$$

Now,

$$f(5) = \frac{a}{3-5} = -\frac{a}{2}$$

$$g(2) = 11 - b \times 2^2$$

$$= 11 - 4b$$

$$\therefore f^2(5) = f \cdot f(5)$$

$$\text{or, } \frac{4}{5} = f\left(-\frac{a}{2}\right)$$

$$\text{or, } \frac{4}{5} = \frac{a}{3 + \frac{a}{2}}$$

$$\text{or, } \frac{4}{5} = \frac{a}{\frac{6+a}{2}}$$

$$\text{or, } \frac{4}{5} = \frac{2a}{6+a}$$

$$\text{or, } 10a = 24 + 4a$$

$$\text{or, } 10a - 4a = 24$$

$$\text{or, } 6a = 24$$

$$\therefore a = 4$$

Again,

$$fg(2) = f(11 - 4b)$$

$$\text{or, } \frac{1}{2} = \frac{a}{3 - (11 - 4b)}$$

$$\text{or, } \frac{1}{2} = \frac{a}{3 - 11 + 4b}$$

$$\text{or, } \frac{1}{2} = \frac{4}{4b - 8}$$

$$\text{or, } 4b - 8 = 8$$

$$\text{or, } 4b = 8 + 8$$

$$\text{or, } 4b = 16$$

$$\therefore b = 4$$

Q. 16 Solⁿ

(a) Here,

$$p(x) = \frac{1}{(x+2)^3} \text{ where, } x \neq -2$$

$$\text{or, } p(x) = (f \circ g)(x)$$

Now,

$$p(x) = \frac{1}{(x+2)^3}$$

$$\text{or, } f \circ g(x) = \frac{1}{(x+2)^3}$$

$$\text{or, } f(g(x)) = \frac{1}{(x+2)^3}$$

$$\text{Let, } g(x) = \frac{1}{x+2} \text{ then,}$$

$$f\left(\frac{1}{x+2}\right) = \left(\frac{1}{x+2}\right)^3$$

$$\therefore f(x) = x^3$$

$$\therefore g(x) = \frac{1}{x+2}$$

(b) Here,
 $h(x) = (3x+2)^3$
 $h(x) = (f \circ g)(x)$

Now,
 $h(x) = (3x+2)^3$

or, $f \circ g(x) = (3x+2)^3$

or, $f(g(x)) = (3x+2)^3$

Let, $g(x) = 3x+2$ then,

$$f(3x+2) = (3x+2)^3$$

$$\therefore f(x) = x^3$$

$$\therefore g(x) = 3x+2$$

Q. N. 17 Solⁿ.

(a) Here,

$$f(x) = \frac{2x+3}{7}$$

$$g(x) = \frac{7x-3}{2}$$

Now,

$$f \circ g(x) = f(g(x))$$

$$= f\left(\frac{7x-3}{2}\right)$$

$$= \frac{2 \times \frac{7x-3}{2} + 3}{7}$$

$$= \frac{7x-3+3}{7}$$

$$= \frac{7x}{7}$$

$$= x$$

$$\therefore f \circ g(x) = x$$

Hence, $f \circ g(x)$ is an identity function.

proved.

(b) Here,

$$f(x) = \frac{4x+5}{9}$$

$$g(x) = \frac{9x-5}{4}$$

Now,

$$f \circ g(x) = f(g(x))$$

$$= f\left(\frac{9x-5}{4}\right)$$

$$= \frac{4 \times \left(\frac{9x-5}{4}\right) + 5}{9}$$

$$= \frac{9x}{9}$$

$$= x$$

$$\therefore f \circ g(x) = x$$

Hence, $f \circ g(x)$ is an identity function.

(c) Here,

$$f(x) = \frac{4x+5}{7x+3}$$

$$g(x) = 15$$

Now,

$$\begin{aligned} f \circ g(x) &= f(g(x)) \\ &= f(15) \\ &= \frac{4 \times 15 + 5}{7 \times 15 + 3} \\ &= \frac{65}{108} \end{aligned}$$

Also,

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g\left(\frac{4x+5}{7x+3}\right) \\ &= 15 \end{aligned}$$

Hence, $f \circ g(x)$ and $g \circ f(x)$ both are constant function.
proved.

(d) Here,

$$f = \{(2,3), (5,6), (7,8)\}$$

$$g = \{(3,9), (6,9), (8,9)\}$$

$$\therefore g \circ f = \{(2,9), (5,9), (7,9)\}$$

where,

$$x = \{2, 5, 7\} \text{ and}$$

$$y = \{9\}$$

$$\therefore y = f(x) = 9$$

Hence, $g \circ f$ is a constant function.
proved.

(e) Here,

$$f(x) = 2x + 3$$

$$g(x) = x^2 + 2x + 1$$

Now,

$$g \circ f(x) = g(f(x))$$

$$= g(2x+3)$$

$$= (2x+3)^2 + 2(2x+3) + 1$$

$$= 4x^2 + 12x + 9 + 4x + 6 + 1$$

$$= 4x^2 + 16x + 16$$

Hence, $g \circ f(x)$ is a form of $ax^2 + bx + c$,
which is quadratic function.
proved.

(B) Here,

$$f(x) = x^3 + x^2 + 1$$

$$g(x) = x - 1$$

Now,

$$\begin{aligned} g \circ f(x) &= g(f(x)) \\ &= g(x^3 + x^2 + 1) \\ &= (x^3 + x^2 + 1) - 1 \\ &= x^3 + x^2 \end{aligned}$$

Hence, $g \circ f(x)$ is a form of $ax^3 + bx^2 + cx + d$, which is cubic function.
proved.

Q.18 Soln.

(a) Here,

$$f = \{(2, 5), (3, 6), (4, 7)\}$$

$$g = \{(5, 12), (6, 14), (7, 16)\}$$

Now,

The notation of the composite function from A to C is $g \circ f: A \rightarrow C$

(b) Here,

$$g \circ f = \{(2, 12), (3, 14), (4, 16)\}$$

(c) Here,

$$f(x) = x + 3$$

$$g(x) = 2x + 2$$

$$\begin{aligned} (g \circ f)(x) &= g \circ f(x) \\ &= g(x+3) \\ &= 2(x+3) + 2 \\ &= 2x + 6 + 2 \\ &= 2x + 8 \end{aligned}$$

$$\therefore (g \circ f)(8) = 2 \times 8 + 8 = 24$$

(d) Here,

$$(g \circ f)(x) = 98$$

$$\text{or, } 2x + 8 = 98$$

$$\text{or, } 2x = 98 - 8$$

$$\text{or, } 2x = 90$$

$$\therefore x = 45$$

Thus, the value of x is 45.