

Exercise - 1.2

Q.1 Solⁿ.

(a) Here,

$$f = \{(1, 8), (2, 9), (3, 10)\}$$

$$\therefore f^{-1} = \{(8, 1), (9, 2), (10, 3)\}$$

(b) Here,

$$g = \{(8, 5), (9, 6), (10, 7)\}$$

$$\therefore g^{-1} = \{(5, 8), (6, 9), (7, 10)\}$$

(c) Here,

$$h = \{(2, 5), (3, 6), (4, 7)\}$$

$$\therefore h^{-1} = \{(5, 2), (6, 3), (7, 4)\}$$

(d) Here,

$$\pi = \{(3, 2), (5, 4), (7, 6)\}$$

$$\therefore \pi^{-1} = \{(2, 3), (4, 5), (6, 7)\}$$

Q.2 Solⁿ.

(a) Here,

$$d = \{(5, 4), (7, 6), (9, 8)\}$$

$$\therefore d^{-1} = \{(4, 5), (6, 7), (8, 9)\}$$

(b) Here,

$$f = \{(1, 3), (2, 4), (3, 5)\}$$

$$\therefore f^{-1} = \{(3, 1), (4, 2), (5, 3)\}$$

Q.3. Solⁿ.

(a) Here,

$$g = \{(0, 2), (1, 3), (2, 5)\}$$

$$\therefore g^{-1}\{3, 5\} = \{1, 2\}$$

(b) Here,

$$i = \{(5, 3), (6, 4), (7, 5)\}$$

$$\therefore i^{-1}\{3\} = \{5\}$$

Q.4 Solⁿ.

(a) Here,

$$f(x) = y = x \text{ and } x = \{1, 2, 3\}$$

$$\text{When } x = 1 \text{ then } y = 1$$

$$\text{When } x = 2 \text{ then } y = 2$$

$$\text{When } x = 3 \text{ then } y = 3$$

$$\Rightarrow f = \{(1, 1), (2, 2), (3, 3)\}$$

$$\therefore f^{-1} = \{(1, 1), (2, 2), (3, 3)\}$$

(b) Here,

Here,

$$f(x) = y = 2x + 1, x = \{0, 1, 2, 3\}$$

$$\text{When } x = 0 \text{ then } y = 2 \times 0 + 1 = 1$$

$$\text{When } x = 1 \text{ then } y = 2 \times 1 + 1 = 3$$

When $x=2$ then, $y = 2 \times 2 + 1 = 5$

When $x=3$ then, $y = 2 \times 3 + 1 = 7$

$$\Rightarrow f = \{(0, 1), (1, 3), (2, 5), (3, 7)\}$$

$$\therefore f^{-1} = \{(1, 0), (3, 1), (5, 2), (7, 3)\}$$

Q.5. Soln.

(a) Here,

$$f = \{(1, 2), (2, 3), (4, 5)\}$$

$$\therefore f^{-1} = \{(2, 1), (3, 2), (5, 4)\}$$

Now,

$$\text{Domain of } f^{-1} = \{2, 3, 5\}$$

$$\text{Range of } f^{-1} = \{1, 2, 4\}$$

(b) Here,

$$f = \{(2, 4), (3, 6), (4, 8)\}$$

$$\therefore f^{-1} = \{(4, 2), (6, 3), (8, 4)\}$$

Now,

$$\text{Domain of } f^{-1} = \{4, 6, 8\}$$

$$\text{Range of } f^{-1} = \{2, 3, 4\}$$

Q.6. (a) Here,

$$f = \{(1, 5), (2, 4), (3, 6)\}$$

$$f \circ g = \{(5, 6), (4, 5), (6, 4)\}$$

Now,

$$f \circ g = f(g(x))$$

$$\Rightarrow f(g(5)) = 6 \text{ and } f(3) = 6$$

$$\therefore g(5) = 3$$

$$\Rightarrow f(g(4)) = 5 \text{ and } f(1) = 5$$

$$\therefore g(4) = 1$$

$$\Rightarrow f(g(6)) = 4 \text{ and } f(2) = 4$$

$$\therefore g(6) = 2$$

$$\text{Thus, } g = \{(5, 3), (4, 1), (6, 2)\}$$

$$\therefore g^{-1} = \{(3, 5), (1, 4), (2, 6)\}$$

(b) Here,

$$f = \{(2, 3), (4, 5), (6, 7)\}$$

$$g \circ f = \{(2, 4), (4, 6), (6, 8)\}$$

Now,

$$g \circ f = g(f(x))$$

$$\Rightarrow g(f(2)) = 4 \text{ and } f(2) = 3$$

$$\therefore g(3) = 4$$

$$\Rightarrow g(f(4)) = 6 \text{ and } f(4) = 5$$

$$\therefore g(5) = 6$$

$$\Rightarrow g(f(6)) = 8 \text{ and } f(6) = 7$$

$$\therefore g(7) = 8$$

$$\Rightarrow g = \{(3, 4), (5, 6), (7, 8)\}$$

$$\therefore g^{-1} = \{(4, 3), (6, 5), (8, 7)\}$$

Q. 7 Soln.

(a) Here,

$$f(x) = 2x + 3, x \in \mathbb{N}$$

Now,

$$y = 2x + 3$$

Interchanging x and y , we get

$$x = 2y + 3$$

$$\text{or, } 2y = x - 3$$

$$\text{or, } y = \frac{x - 3}{2}$$

$$\therefore f^{-1}(x) = \frac{x - 3}{2}$$

(b) Here,

$$f(x) = 4x - 5$$

Now,

$$y = 4x - 5$$

Interchanging x and y , we get

$$x = 4y - 5$$

$$\text{or, } x + 5 = 4y$$

$$\text{or, } y = \frac{x+5}{4}$$

$$\therefore f^{-1}(x) = \frac{x+5}{4}$$

(c) Here,

$$f(x) = \frac{x+5}{3}$$

Now,

$$y = \frac{x+5}{3}$$

Interchanging x and y , we get

$$x = \frac{y+5}{3}$$

$$\text{or, } 3x = y + 5$$

$$\text{or, } 3x - 5 = y$$

$$\text{or, } y = 3x - 5$$

$$\therefore f^{-1}(x) = 3x - 5$$

(d) Here,

$$f(x) = \frac{x-3}{4}$$

Now,

$$y = \frac{x-3}{4}$$

Interchanging x and y , we get

$$x = \frac{y-3}{4}$$

$$\text{or, } 4x = y-3$$

$$\text{or, } 4x+3 = y$$

$$\text{or, } y = 4x+3$$

$$\therefore f^{-1}(x) = 4x+3$$

(e) Here,

$$f(x) = \frac{2x+5}{6}$$

$$\text{or, } y = \frac{2x+5}{6}$$

Interchanging x and y , we get

$$x = \frac{2y+5}{6}$$

$$\text{or, } 2y+5 = 6x$$

$$\text{or, } 2y = 6x-5$$

$$\text{or, } y = \frac{6x-5}{5}$$

$$\therefore f^{-1}(x) = \frac{6x-5}{5}$$

(F) Here,

$$g(x) = \frac{3x-5}{7}$$

Now,

$$y = \frac{3x-5}{7}$$

Interchanging x and y , we get

$$x = \frac{3y-5}{7}$$

$$\text{or, } 3y-5 = 7x$$

$$\text{or, } 3y = 7x+5$$

$$\text{or, } y = \frac{7x+5}{3}$$

$$\therefore g^{-1}(x) = \frac{7x+5}{3}$$

(8) Here,

$$y = f(x) = \frac{2x+3}{x-3}$$

Interchanging x and y , we get

$$x = \frac{2y+3}{y-3}$$

$$\text{or, } xy - 3x = 2y + 3$$

$$\text{or, } xy - 2y = 3 + 3x$$

$$\text{or, } y(x-2) = 3+3x$$

$$\text{or, } y = \frac{3+3x}{x-2}$$

$$\therefore f^{-1}(x) = \frac{3x+3}{x-2}$$

(h) Here,

$$y = g(x) = \frac{3x-5}{x+2}$$

Interchanging x and y , we get

$$x = \frac{3y-5}{y+2}$$

$$\text{or, } xy + 2x = 3y - 5$$

$$\text{or, } xy - 3y = -5 - 2x$$

$$\text{or, } y(x-3) = -(2x+5)$$

$$\text{or, } y = \frac{-(2x+5)}{x-3}$$

$$\text{or, } y = \frac{2x+5}{3-x}$$

$$\therefore g^{-1}(x) = \frac{2x+5}{3-x}$$

(i) Here,

$$y = f(x) = \frac{4x+7}{5x+3}$$

Interchanging x and y , we get

$$x = \frac{4y+7}{5y+3}$$

$$\text{or, } 5xy + 3x = 4y + 7$$

$$\text{or, } 5xy - 4y = 7 - 3x$$

$$\text{or, } y(5x - 4) = 7 - 3x$$

$$\text{or, } y = \frac{7 - 3x}{5x - 4}$$

$$\therefore f^{-1}(x) = \frac{7 - 3x}{5x - 4}$$

Q.8 Solⁿ

(a) Here,

$$f(x) = \frac{x-5}{4}$$

$$f^{-1}(x) = 25$$

Now,

$$y = \frac{x-5}{4}$$

Interchanging x and y , we get

$$x = \frac{y-5}{4}$$

$$\text{or, } y-5 = 4x$$

$$\text{or, } y = 4x+5$$

$$\therefore f^{-1}(x) = 4x+5$$

Again,

$$f^{-1}(x) = 25$$

$$\text{or, } 4x+5 = 25$$

$$\text{or, } 4x = 25-5$$

$$\text{or, } 4x = 20$$

$$\text{or, } x = \frac{20}{4}$$

$$\therefore x = 5$$

③

Here,

$$f(x) = 2x - a$$

$$f^{-1}(2) = 2$$

Now,

$$y = 2x - a$$

Interchanging x and y , we get

$$x = 2y - a$$

$$\text{or, } x + a = 2y$$

$$\text{or, } y = \frac{x + a}{2}$$

$$\therefore f^{-1}(x) = \frac{x + a}{2}$$

Again,

$$f^{-1}(2) = \frac{2 + a}{2}$$

$$\text{or, } 2 = \frac{2 + a}{2}$$

$$\text{or, } 2 + a = 4$$

$$\text{or, } a = 4 - 2$$

$$\therefore a = 2$$

①

9. (a) Here,

$$f(4x+5) = 12x+18$$

$$\text{Let, } 4x+5 = z$$

$$\text{or, } 4x = z-5$$

$$\therefore x = \frac{z-5}{4}$$

Then,

$$f\left(4 \times \frac{z-5}{4} + 5\right) = 12 \times \left(\frac{z-5}{4}\right) + 18$$

$$\text{or, } f(z) = 3z - 15 + 18$$

$$\text{or, } f(z) = 3z + 3$$

$$\therefore f(x) = 3x + 3$$

Now,

$$y = 3x + 3$$

Interchanging x and y , we get

$$x = 3y + 3$$

$$\text{or, } x-3 = 3y$$

$$\text{or, } y = \frac{x-3}{3}$$

$$\therefore f^{-1}(x) = \frac{x-3}{3}$$

(b)

Here,

$$f(x+3) = 3x+5$$

$$\text{Let, } x+3 = z$$

$$\therefore x = z-3$$

Then,

$$f(z-3+3) = 3 \times (z-3) + 5$$

$$\text{or, } f(z) = 3z - 9 + 5$$

$$\text{or, } f(z) = 3z - 4$$

$$\therefore f(x) = 3x - 4$$

Again,

$$y = 3x - 4$$

Interchanging x and y , we get

$$x = 3y - 4$$

$$\text{or, } x + 4 = 3y$$

$$\text{or, } y = \frac{x+4}{3}$$

$$\therefore f^{-1}(x) = \frac{x+4}{3}$$

~~Q. 10~~ Solⁿ

(a) Here,

$$f(x) = 2x + 7$$

$$\begin{aligned}\therefore f(x+2) &= 2 \times (x+2) + 7 \\ &= 2x + 4 + 7 \\ &= 2x + 11\end{aligned}$$

Also,

$$y = 2x + 7$$

Interchanging x and y , we get

$$x = 2y + 7$$

$$\text{or, } x - 7 = 2y$$

$$\text{or, } y = \frac{x-7}{2}$$

$$\therefore f^{-1}(x) = \frac{x-7}{2}$$

Now,

$$\begin{aligned}\Rightarrow f^{-1}(x+2) &= \frac{x+2-7}{2} \\ &= \frac{x-5}{2}\end{aligned}$$

(b) Here,

$$f(x) = 4 - 3x$$

$$\begin{aligned}\therefore f(x+2) &= 4 - 3x(x+2) \\ &= 4 - 3x - 6 \\ &= -3x - 2\end{aligned}$$

Also,

$$y = 4 - 3x$$

Interchanging x and y , we get

$$x = 4 - 3y$$

$$\text{or, } 3y = 4 - x$$

$$\text{or, } y = \frac{4-x}{3}$$

$$\therefore f^{-1}(x) = \frac{4-x}{3}$$

Now,

$$f^{-1}(x+2) = \frac{4-(x+2)}{3}$$

$$= \frac{4-x-2}{3}$$

$$= \frac{2-x}{3}$$

Q.11 Soln

(a) Here,

$$f(x) = 2x + 1$$

$$g(x) = \frac{x-5}{2}$$

Now

$$f(x) = 2x + 1$$

$$\text{or, } y = 2x + 1$$

Interchanging x and y , we get

$$x = 2y + 1$$

$$\text{or, } x - 1 = 2y$$

$$\text{or, } y = \frac{x-1}{2}$$

$$\therefore f^{-1}(x) = \frac{x-1}{2}$$

Also,

$$g(x) = \frac{x-5}{2}$$

$$y = \frac{x-5}{2}$$

Interchanging x and y we get

$$x = \frac{y-5}{2}$$

$$\text{or, } y - 5 = 2x$$

$$\text{or, } y = 2x + 5$$

$$\therefore g^{-1}(x) = 2x + 5$$

$$g^{-1}(3) = 2 \times 3 + 5 = 6 + 5 = 11$$

Now,

$$f^{-1}g^{-1}(3) = f^{-1}(11)$$

$$= \frac{11-1}{2}$$

$$= \frac{10}{2}$$

$$= 5$$

(b) Here,

$$3f(x) = 4x + 5$$

$$\text{or, } f(x) = \frac{4x+5}{3}$$

$$\text{or, } y = \frac{4x+5}{3}$$

Interchanging x and y , we get

$$x = \frac{4y+5}{3}$$

$$4y + 5 = 3x$$

$$\text{or, } 4y = 3x - 5$$

$$\therefore y = \frac{3x-5}{4}$$

$$f^{-1}(x) = \frac{3x-5}{4}$$

Also,

$$g(x) = 5x - 4$$

$$y = 5x - 4$$

Interchanging x and y , we get

$$x = 5y - 4$$

$$\text{or, } x + 4 = 5y$$

$$\text{or, } y = \frac{x+4}{5}$$

$$\therefore g^{-1}(x) = \frac{x+4}{5}$$

$$\Rightarrow g^{-1}(1) = \frac{1+4}{5} = \frac{5}{5} = 1$$

Now,

$$f^{-1} \circ g^{-1}(1) = f^{-1}(g^{-1}(1))$$

$$= f^{-1}(1)$$

$$= \frac{3 \times 1 - 5}{4}$$

$$= \frac{3-5}{4}$$

$$= -\frac{2}{4}$$

$$= -\frac{1}{2}$$

Q.12 Solⁿ.

(a) Here,

$$f(x) = 1 + 2x$$

$$g(x) = \frac{1}{1-x}, x \neq 1$$

Now,

$$y = g(x) = \frac{1}{1-x}$$

Interchanging x and y , we get

$$x = \frac{1}{1-y}$$

$$\text{or, } x - xy = 1$$

$$\text{or, } x - 1 = xy$$

$$\text{or, } y = \frac{x-1}{x}$$

$$\therefore g^{-1}(x) = \frac{x-1}{x}$$

$$\Rightarrow g^{-1}\left(\frac{1}{2}\right) = \frac{\frac{1}{2} - 1}{\frac{1}{2}}$$

$$= -\frac{1}{2} \times \frac{2}{1}$$

$$= -1$$

Again,

$$\begin{aligned}
 g(-1) &= \frac{1}{1 - (-1)} \\
 &= \frac{1}{1 + 1} \\
 &= \frac{1}{2}
 \end{aligned}$$

Now,

$$\begin{aligned}
 fog(-1) &= f(g(-1)) \\
 &= f\left(\frac{1}{2}\right) \\
 &= 1 + 2 \times \frac{1}{2} \\
 &= 1 + 1 \\
 &= 2
 \end{aligned}$$

(b) Here,

$$f(x) = \frac{2x+3}{x+2}$$

$$g(x) = x-2$$

Now,

$$y = f(x) = \frac{2x+3}{x+2}$$

Interchanging x and y , we get

$$x = \frac{2y+3}{y+2}$$

$$\text{or, } xy + 2x = 2y + 3$$

$$\text{or, } xy - 2y = 3 - 2x$$

$$\text{or, } y(x-2) = 3-2x$$

$$\text{or, } y = \frac{3-2x}{x-2}$$

$$\therefore f^{-1}(x) = \frac{3-2x}{x-2}$$

Also,

$$f^{-1}(1) = \frac{3-2 \times 1}{1-2}$$

$$= \frac{3-2}{-1}$$

$$= \frac{1}{-1}$$

$$= -1$$

Also,

$$fg(x) = f(x-2)$$

$$= \frac{2(x-2)+3}{(x-2)+2}$$

$$= \frac{2x-4+3}{x-2+2}$$

$$= \frac{2x-1}{x}$$

$$= \frac{2x-1}{x}$$

Also,

$$fg(1) = f(1-2)$$

$$= f(-1)$$

$$= \frac{2 \times (-1) + 3}{-1 + 2}$$

$$= \frac{-2+3}{1}$$

$$= \frac{-2+3}{1}$$

$$= 1$$

Q.13 Solⁿ

(a) Here,

$$f(x) = 3x+4$$

$$g(x) = 2(x+1) = 2x+2$$

Now,

$$\begin{aligned} \text{L.H.S.} &= f \circ g \\ &= f(g(x)) \\ &= f(2x+2) \\ &= 3(2x+2)+4 \\ &= 6x+6+4 \\ &= 6x+10 \end{aligned}$$

$$\begin{aligned} \text{R.H.S.} &= g \circ f \\ &= g(f(x)) \\ &= g(3x+4) \\ &= 2(3x+4)+2 \\ &= 6x+8+2 \\ &= 6x+10 \end{aligned}$$

Hence, $f \circ g = g \circ f = 6x+10$.
proved.

Again,

$$y = f(x) = 3x + 4$$

Interchanging x and y , we get

$$x = 3y + 4$$

$$\text{or, } x - 4 = 3y$$

$$\text{or, } y = \frac{x - 4}{3}$$

$$\therefore f^{-1}(x) = \frac{x - 4}{3}$$

Now,

$$f^{-1}(2) = \frac{2 - 4}{3}$$

$$= -\frac{2}{3}$$

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(b) Here

$$f(x) = 2x - 3$$

$$\text{or, } y = 2x - 3$$

Interchanging x and y , we get

$$x = 2y - 3$$

$$\text{or, } x + 3 = 2y$$

$$\text{or, } y = \frac{x + 3}{2}$$

$$\therefore f^{-1}(x) = \frac{x + 3}{2}$$

Now,

$$\text{L.H.S.} = f f^{-1}(x)$$

$$= f\left(\frac{x+3}{2}\right)$$

$$= 2 \times \left(\frac{x+3}{2}\right) - 3$$

$$= x+3-3$$

$$= x$$

$$\begin{aligned} \text{R.H.S.} &= f^{-1} f(x) \\ &= f^{-1}(2x-3) \end{aligned}$$

$$= \frac{2x-3+3}{2}$$

$$= \frac{2x}{2}$$

$$= x$$

$$\therefore f f^{-1}(x) = f^{-1} f(x) = x$$

Hence, proved.

Q. 14. Solⁿ

(a) Here,

$$f(x) = 3x + a$$

$$ff(6) = 10$$

Now,

$$ff(6) = f(3 \times 6 + a)$$

$$\text{or, } 10 = f(18 + a)$$

$$\text{or, } 10 = 3(18 + a) + a$$

$$\text{or, } 10 = 54 + 3a + a$$

$$\text{or, } 10 - 54 = 4a$$

$$\text{or, } 4a = -44$$

$$\therefore a = -11$$

$$\text{So, } f(x) = 3x - 11$$

$$y = 3x - 11$$

Interchanging x and y , we get

$$x = 3y - 11$$

$$\text{or, } x + 11 = 3y$$

$$\text{or, } y = \frac{x + 11}{3}$$

$$\therefore f^{-1}(x) = \frac{x + 11}{3}$$

Also,

$$f^{-1}(4) = \frac{4 + 11}{3}$$

$$= \frac{15}{3}$$

$$= 5$$

(b) Here,

$$f(x) = \frac{x}{2x - 3}$$

$$\text{or, } y = \frac{x}{2x - 3}$$

Interchanging x and y , we get

$$x = \frac{y}{2y - 3}$$

$$\text{or, } 2xy - 3x = y$$

$$\text{or, } 2xy - y = 3x$$

$$\text{or, } y(2x - 1) = 3x$$

$$\text{or, } y = \frac{3x}{2x-1}$$

$$\therefore f^{-1}(x) = \frac{3x}{2x-1}$$

We have,

$$f(x) = f^{-1}(x)$$

$$\frac{x}{2x-3} = \frac{3x}{2x-1}$$

$$\text{or, } 2x^2 - x = 6x^2 - 9x$$

$$\text{or, } 6x^2 - 2x^2 - 9x + x = 0$$

$$\text{or, } 4x^2 - 8x = 0$$

$$\text{or, } 4x(x - 2) = 0$$

Either,

$$\Rightarrow 4x = 0$$

$$\therefore x = 0$$

$$\Rightarrow x - 2 = 0$$

$$\therefore x = 2$$

Thus,

the values of x are 0 or 2.

Q.15 Solⁿ:

(a) Here,

$$f(x) = 4x + 7$$

$$g(x) = 3x - 5$$

$$fg^{-1}(x) = 15$$

Now,

$$y = g(x) = 3x - 5$$

Interchanging x and y , we get

$$x = 3y - 5$$

$$\text{or, } x + 5 = 3y$$

$$\text{or, } y = \frac{x+5}{3}$$

$$\therefore g^{-1}(x) = \frac{x+5}{3}$$

Again,

$$fg^{-1}(x) = f\left(\frac{x+5}{3}\right)$$

$$\text{or, } 15 = 4\left(\frac{x+5}{3}\right) + 7$$

$$\text{or, } 15 = \frac{4x+20}{3} + 7$$

$$\text{or, } 15 = \frac{4x+20+21}{3}$$

$$\text{or, } 45 = 4x + 41$$

$$\text{or, } 45 - 41 = 4x$$

$$\text{or, } 4 = 4x$$

$$\therefore x = 1$$

(b) Here,

$$f(x) = x^2 - 2x$$

$$g(x) = 2x + 3$$

$$f(g^{-1}(x)) = 3$$

Now,

$$y = g(x) = 2x + 3$$

Interchanging x and y , we get

$$x = 2y + 3$$

$$\text{or, } x - 3 = 2y$$

$$\text{or, } y = \frac{x-3}{2}$$

$$\therefore g^{-1}(x) = \frac{x-3}{2}$$

Again,

$$f(g^{-1}(x)) = f\left(\frac{x-3}{2}\right)$$

$$\text{or, } 3 = \left(\frac{x-3}{2}\right)^2 - 2 \times \left(\frac{x-3}{2}\right)$$

$$\text{or, } 3 = \frac{x^2 - 6x + 9}{4} - x + 3$$

$$\text{or, } 3 = \frac{x^2 - 6x + 9 - 4x + 12}{4}$$

$$\text{or, } 12 = x^2 - 10x + 21$$

$$\text{or, } 0 = x^2 - 10x + 21 - 12$$

$$\text{or, } x^2 - 10x + 9 = 0$$

$$\text{or, } x^2 - (9+1)x + 9 = 0$$

$$\text{or, } x^2 - 9x - 1x + 9 = 0$$

$$\text{or, } x(x-9) - 1(x-9) = 0$$

$$\text{or, } (x-9)(x-1) = 0$$

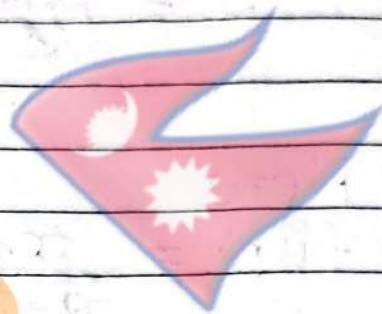
Either,

$$\Rightarrow x-9=0 \quad | \quad \Rightarrow x-1=0$$

$$\therefore x=9$$

$$\therefore x=1$$

Hence, $x = 9$ or 1



(c) Here,

$$f(x) = 4x - 17$$

$$g(x) = \frac{2x+8}{5}$$

Now,

$$y = g(x) = \frac{2x+8}{5}$$

Interchanging x and y , we get

$$x = \frac{2y+8}{5}$$

$$\text{or, } 5x = 2y + 8$$

$$\text{or, } 5x - 8 = 2y$$

$$\text{or, } y = \frac{5x-8}{2}$$

$$\therefore g^{-1}(x) = \frac{5x-8}{2}$$

We have,

$$f(x) = g^{-1}(x)$$

$$\text{or, } 4x - 17 = \frac{5x-8}{2}$$

$$\text{or, } 8x - 34 = 5x - 8$$

$$\text{or, } 8x - 5x = 34 - 8$$

$$\text{or, } 3x = 26$$

$$\therefore x = \frac{26}{3}$$

(d) Here,

$$f(x) = \frac{x}{2-x}, \quad x \neq 2$$

$$g(x) = 6x - 2$$

Now,

$$y = f(x) = \frac{x}{2-x}$$

Interchanging x and y , we get,

$$x = \frac{y}{2-y}$$

$$\text{or, } 2x - xy = y$$

$$\text{or, } 2x = y + xy$$

$$\text{or, } 2x = (1+x)y$$

$$\text{or, } y = \frac{2x}{x+1}$$

$$\therefore f^{-1}(x) = \frac{2x}{x+1}$$

$$\Rightarrow g f(4) = g\left(\frac{4}{2-4}\right)$$

$$\text{or, } -8 = g\left(-\frac{4}{2}\right)$$

$$\text{or, } -8 = g(-2)$$

$$\text{or, } -8 = b \times (-2) - 2$$

$$\text{or, } -8 = -2b - 2$$

$$\text{or, } -8 + 2 = -2b$$

$$\text{or, } -6 = -2b$$

$$\therefore b = 3$$

Also,

$$f^{-1}(-2) = \frac{2 \times (-2)}{-2 + 1}$$

$$= \frac{-4}{-1}$$

$$= 4$$

(e) Here,

$$f(x) = \frac{3x+11}{x-3}, x \neq 3$$

$$g(x) = \frac{x-3}{2}$$

Now,

$$y = f(x) = \frac{3x+11}{x-3}$$

Interchanging x and y , we get

$$x = \frac{3y+11}{y-3}$$

$$\text{or, } xy - 3x = 3y + 11$$

$$\text{or, } xy - 3y = 11 + 3x$$

$$\text{or, } y(x-3) = 11 + 3x$$

$$\text{or, } y = \frac{11 + 3x}{x-3}$$

$$\therefore f^{-1}(x) = \frac{11 + 3x}{x-3} = \frac{3x + 11}{x-3}$$

Again,

$$y = g(x) = \frac{x-3}{2}$$

Interchanging x and y , we get

$$x = \frac{y-3}{2}$$

$$\text{or, } 2x = y - 3$$

$$\text{or, } 2x + 3 = y$$

$$\text{or, } y = 2x + 3$$

$$\therefore g^{-1}(x) = 2x + 3$$

Now

$$f(x) = g^{-1}(x)$$

$$\text{or, } \frac{3x + 11}{x-3} = 2x + 3$$

$$\text{or, } 3x + 11 = 2x^2 - 6x + 3x - 9$$

$$\text{or, } 3x + 11 = 2x^2 - 3x - 9$$

$$\text{or, } 2x^2 - 3x - 3x - 9 - 11 = 0$$

$$\text{or, } 2x^2 - 6x - 20 = 0$$

$$\text{or, } x^2 - 3x - 10 = 0$$

$$\text{or, } x^2 - 5x + 2x - 10 = 0$$

$$\text{or, } x(x-5) + 2(x-5) = 0$$

$$\text{or, } (x-5)(x+2) = 0$$

Either,

$$\Rightarrow x-5 = 0$$

$$\therefore x = 5$$

$$\Rightarrow x+2 = 0$$

$$\therefore x = -2$$

Hence, $x = -2$ or 5

Q.16 Solⁿ.

(a) Here,

$$f(x) = \frac{3x+5}{x-2}$$

$$g(x) = 7$$

Now,

$$y = f(x) = \frac{3x+5}{x-2}$$

Interchanging x and y , we get

$$x = \frac{3y+5}{y-2}$$

$$\text{or, } xy - 2x = 3y + 5$$

$$\text{or, } xy - 3y = 5 + 2x$$

$$\text{or, } y(x-3) = 2x+5$$

$$\text{or, } y = \frac{2x+5}{x-3}$$

$$\therefore f^{-1}(x) = \frac{2x+5}{x-3}$$

Again,

$$f^{-1} \circ g(x) = f^{-1}(g(x))$$

$$= f^{-1}(7)$$

$$= \frac{2 \times 7 + 5}{7 - 3}$$

$$= \frac{14 + 5}{4}$$

$$= \frac{19}{4}$$

Hence, $f^{-1} \circ g(x) = \frac{19}{4}$ is a constant function.

(b) Here,

$$h(x) = \frac{2x - 3}{7}$$

$$f(x) = 3$$

Now,

$$y = h(x) = \frac{2x - 3}{7}$$

Interchanging x and y , we get

$$x = \frac{2y - 3}{7}$$

$$\text{or, } 2y - 3 = 7x$$

$$\text{or, } 2y = 7x + 3$$

$$\text{or, } y = \frac{7x + 3}{2}$$

$$\therefore h^{-1}(x) = \frac{7x+3}{2}$$

Again,

$$h^{-1} \circ f(x) = h^{-1}(f(x))$$

$$= h^{-1}(3)$$

$$= \frac{7 \times 3 + 3}{2}$$

$$= \frac{21 + 3}{2}$$

$$= \frac{24}{2}$$

$$= 12$$

Hence, $h^{-1} \circ f(x) = 12$ is a constant function.

Q.17. Soln.

(a) Here,

$$f(x) = 2x + 3$$

$$g(x) = x^2 + 2x + 1$$

Now,

$$y = f(x) = 2x + 3$$

Interchanging x and y , we get

$$x = 2y + 3$$

$$\text{or, } x - 3 = 2y$$

$$\text{or, } y = \frac{x-3}{2}$$

$$\therefore f^{-1}(x) = \frac{x-3}{2}$$

Again,

$$\begin{aligned} f^{-1} \circ g(x) &= f^{-1}g(x) \\ &= f^{-1}(x^2 + 2x + 1) \end{aligned}$$

$$= \frac{x^2 + 2x + 1 - 3}{2}$$

$$= \frac{x^2 + 2x - 2}{2}$$

$$= \frac{1}{2}x^2 + \frac{2x}{2} - \frac{2}{2}$$

$$= \frac{1}{2}x^2 + x - 1$$

Hence, $f^{-1} \circ g(x)$ is a quadratic function.
proved.

(6) Here,

$$p(x) = x + 1$$

$$q(x) = x^2 + 2$$

Now,

$$y = p(x) = x + 1$$

Interchanging x and y , we get

$$x = y + 1$$

$$\therefore y = x - 1$$

$$\therefore p^{-1}(x) = x - 1$$

$$\begin{aligned}
 q \circ p^{-1}(x) &= q \circ p^{-1}(x) \\
 &= q(x-1) \\
 &= (x-1)^2 + 2 \\
 &= x^2 - 2x + 1 + 2 \\
 &= x^2 - 2x + 3
 \end{aligned}$$

$\therefore q \circ p^{-1}(x)$ is a quadratic function.
Hence, proved.

Q.18. Soln.

(a) Here,

$$f(x) = 3x + 5$$

$$g(x) = \frac{3x+2}{4}$$

Now,

$$y = g(x) = \frac{3x+2}{4}$$

Interchanging x and y , we get

$$x = \frac{3y+2}{4}$$

$$\text{or, } 3y + 2 = 4x$$

$$\text{or, } 3y = 4x - 2$$

$$\text{or, } y = \frac{4x-2}{3}$$

$$\therefore g^{-1}(x) = \frac{4x-2}{3}$$

Again,

$$f \circ g^{-1}(x) = f(g^{-1}(x))$$
$$\text{or, } x = f\left(\frac{4x-2}{3}\right)$$

$$\text{or, } x = 3 \times \left(\frac{4x-2}{3}\right) + 5$$

$$\text{or, } x = 4x - 2 + 5$$

$$\text{or, } x = 4x + 3$$

$$\text{or, } 4x - x = -3$$

$$\text{or, } 3x = -3$$

$$\therefore x = -1$$

(b) Here,

$$f(x) = 4x + 2$$

$$g(x) = \frac{4x+3}{4}$$

Now,

$$y = g(x) = \frac{4x+3}{4}$$

Interchanging x and y , we get

$$x = \frac{4y+3}{4}$$

$$\text{or, } 4y + 3 = 4x$$

$$\text{or, } 4y = 4x - 3$$

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$$y = \frac{4x-3}{4}$$

$$\therefore g^{-1}(x) = \frac{4x-3}{4}$$

Again,

$$f \circ g^{-1}(x) = f(g^{-1}(x))$$

$$\text{or, } x = f\left(\frac{4x-3}{4}\right)$$

$$\text{or, } x = 4\left(\frac{4x-3}{4}\right) + 2$$

$$\text{or, } x = 4x - 3 + 2$$

$$\text{or, } x = 4x - 1$$

$$\text{or, } 4x - x = 1$$

$$\text{or, } 3x = 1$$

$$\therefore x = \frac{1}{3}$$

Q.19. Soln.

(a) Here,

$$f = \{(1,3), (2,5), (3,7)\}$$

$$g = \{(3,8), (5,14), (7,20)\}$$

Now,

$$f^{-1} = \{(3,1), (5,2), (7,3)\}$$

(b) Here,

$$f^{-1}(5) = 2$$

(c) Here,

$$g \circ f = \{(1, 8), (2, 14), (3, 20)\}$$

$$(g \circ f)^{-1} = \{(8, 1), (14, 2), (20, 3)\}$$

(d) Here,

$$(g \circ f)^{-1}(x) = \frac{x-2}{6}$$

$$\text{or, } 40 = \frac{x-2}{6}$$

$$\text{or, } 240 + 2 = x$$

$$\therefore x = 242$$