

Chapter 2 polynomial

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Exercise 2.1

Q.1. Solⁿ:

(a) Here,

$$f(x) = x^2 + 3x + 12$$

$$g(x) = x + 1$$

Now,

$$x + 1 = 0$$

$$\therefore x = -1$$

Using remainder theorem,

$$\begin{aligned} R &= f(-1) \\ &= (-1)^2 + 3(-1) + 12 \\ &= 1 - 3 + 12 \\ &= 10 \end{aligned}$$

Thus, the remainder is 10.

(b) Here,

$$f(x) = x^5 - x^3 + 20$$

$$g(x) = x + 2$$

Now,

$$x + 2 = 0$$

$$\therefore x = -2$$

Using remainder theorem,

$$\begin{aligned} R &= f(-2) \\ &= (-2)^5 - (-2)^3 + 20 \\ &= -32 - (-8) + 20 \\ &= -32 + 8 + 20 \\ &= -4 \end{aligned}$$

Thus, the remainder is -4.

(c) Here,

$$f(x) = 2x^5 - 3x^2 - x + 5$$

$$g(x) = x - 4$$

Now,

$$x - 4 = 0$$

$$\therefore x = 4$$

Using remainder theorem,

$$R = f(4)$$

$$= 2 \times 4^5 - 3 \times 4^2 - 4 + 5$$

$$= 2 \times 1024 - 3 \times 16 - 4 + 5$$

$$= 2001$$

(d) Here,

$$f(x) = 3x^3 - 5x^2 + 2x - 3$$

$$g(x) = x - 2$$

Now,

$$x - 2 = 0$$

$$\therefore x = 2$$

$$R = f(2)$$

$$= 3 \times 2^3 - 5 \times 2^2 + 2 \times 2 - 3$$

$$= 3 \times 8 - 5 \times 4 + 4 - 3$$

$$= 24 - 20 + 4 - 3$$

$$= 5$$

Soln.

$$(e) f(x) = x^3 - Kx^2 - 2x + K + 4$$

$$g(x) = x - K$$

Now,

$$x - K = 0$$

$$\therefore x = K$$

Using remainder theorem,

$$R = f(K)$$

$$= K^3 - K \times K^2 - 2 \times K + K + 4$$

$$= K^3 - K^3 - 2K + K + 4$$

$$= 4 - K$$

Thus, the remainder is $4 - K$.

(f) Here,

$$f(x) = 2x^2 + 15x + 7$$

$$g(x) = 2x + 1$$

Now,

$$2x + 1 = 0$$

$$\therefore 2x = -1$$

$$\therefore x = -\frac{1}{2}$$

Using remainder theorem,

$$R = f\left(-\frac{1}{2}\right)$$

$$= 2 \times \left(-\frac{1}{2}\right)^2 + 15 \times \left(-\frac{1}{2}\right) + 7$$

$$= 2 \times \frac{1}{4} - \frac{15}{2} + 7$$

$$= \frac{1}{2} - \frac{15}{2} + 7$$

$$= \frac{1 - 15 + 14}{2}$$

$$= \frac{0}{2}$$

$$= 0$$

Thus, the remainder is 0.

(g) Here,

$$f(x) = 2x^3 - 11x^2 + 19x - 10$$

$$g(x) = 2x - 5$$

Now,

$$2x - 5 = 0$$

$$\therefore x = \frac{5}{2}$$

Using remainder theorem,

$$\text{Remainder} = f\left(\frac{5}{2}\right)$$

$$= 2 \times \left(\frac{5}{2}\right)^3 - 11 \times \left(\frac{5}{2}\right)^2 + 19 \times \frac{5}{2} - 10$$

$$= 2 \times \frac{125}{8} - 11 \times \frac{25}{4} + \frac{95}{2} - 10$$

$$= \frac{125}{4} - \frac{275}{4} + \frac{95}{2} - 10$$

$$= \frac{125 - 275 + 2 \times 95 - 4 \times 10}{4}$$

$$= \frac{0}{4}$$

$$= 0$$

Thus,
the remainder is 0.

Q.2 Solⁿ

Here,

$$f(y) = y^3 - ay^2 - 2y + 1$$

Now,

$$y - a = 0$$

$$\therefore y = a$$

Then,

a	1	-a	-2	1
		a	0	-2a
	1	0	-2	1-2a

$$\therefore \text{Remainder} = 1 - 2a$$

Again,

$$\begin{aligned} f(a) &= a^3 - a \times a^2 - 2 \times a + 1 \\ &= a^3 - a^3 - 2a + 1 \\ &= 1 - 2a \text{ equal} \end{aligned}$$

Thus, we see the relation between remainder and $f(a)$.

Q23. (a) Here,

$$f(x) = 4x^3 - 12x^2 + 11x - 3$$

It is divided by $(x - \frac{1}{2})$,

$$\text{So, } x = \frac{1}{2}$$

Using remainder theorem,

$$R = f\left(\frac{1}{2}\right)$$

$$= 4 \times \left(\frac{1}{2}\right)^3 - 12 \times \left(\frac{1}{2}\right)^2 + 11 \times \frac{1}{2} - 3$$

$$= 4 \times \frac{1}{8} - 12 \times \frac{1}{4} + \frac{11}{2} - 3$$

$$= \frac{1}{2} - 3 + \frac{11}{2} - 3$$

$$= \frac{1 - 6 + 11 - 6}{2}$$

$$= \frac{0}{2}$$

$$= 0$$

Thus,

the remainder is 0.

(b) Here,

$$f(x) = 4x^3 - 12x^2 + 11x - 3$$

It is divided by $(x + \frac{1}{2})$,

$$\text{So, } x = -\frac{1}{2}$$

Using remainder theorem,

$$R = f\left(-\frac{1}{2}\right)$$

$$= 4 \times \left(-\frac{1}{2}\right)^3 - 12 \times \left(-\frac{1}{2}\right)^2 + 11 \times \left(-\frac{1}{2}\right) - 3$$

$$= -4 \times \frac{1}{8} - 12 \times \frac{1}{4} - \frac{11}{2} - 3$$

$$= -\frac{1}{2} - 3 - \frac{11}{2} - 3$$

$$= \frac{-1 - 6 - 11 - 6}{2}$$

$$= -\frac{24}{2}$$

$$= -12$$

Thus,

the remainder is -12 .

(c) Here,

$$f(x) = 4x^3 - 12x^2 + 11x - 3$$

It is divided by $(x - \frac{3}{2})$,

$$\text{So, } x = \frac{3}{2}$$

Using remainder theorem,

$$R = f\left(\frac{3}{2}\right)$$

$$= 4 \times \left(\frac{3}{2}\right)^3 - 12 \times \left(\frac{3}{2}\right)^2 + 11 \times \frac{3}{2} - 3$$

$$= 4 \times \frac{27}{8} - 12 \times \frac{9}{4} + \frac{33}{2} - 3$$

$$= \frac{27}{2} - 3 \times 9 + \frac{33}{2} - 3$$

$$= \frac{27}{2} - 30 + \frac{33}{2}$$

$$= \frac{27 - 60 + 33}{2}$$

$$= \frac{0}{2}$$

$$= 0$$

Q.4. Soln.

(a) Here,

$$P(x) = 2x^3 + 3x^2 + Kx + 4$$

$$R = 2(4 - K) = 8 - 2K$$

$$D(x) = x - 2$$

Now,

$$x - 2 = 0$$

$$\therefore x = 2$$

Using remainder theorem,

$$R = P(2)$$

$$\text{or } 8 - 2K = 2 \times 2^3 + 3 \times 2^2 + K \times 2 + 4$$

$$\text{or } 8 - 2K = 16 + 12 + 2K + 4$$

$$\text{or } 8 - 2K = 32 + 2K$$

$$\text{or } -2K - 2K = 32 - 8$$

$$\text{or } -4K = 24$$

$$\text{or } K = \frac{24}{-4}$$

$$\therefore K = -6$$

Thus,

the value of K is -6 .

(b) Here,

$$p(x) = x^3 - ax^2 - 2x + 2a + 6$$

$$R = a + 2$$

$$Q(x) = x - a$$

Now,

$$x - a = 0$$

$$\therefore x = a$$

Using remainder theorem,

$$R = p(a)$$

$$\text{or, } a + 2 = a^3 - a \cdot a^2 - 2 \cdot a + 2a + 6$$

$$\text{or, } a + 2 = a^3 - a^3 - 2a + 2a + 6$$

$$\text{or, } a + 2 = 6$$

$$\text{or, } a = 6 - 2$$

$$\therefore a = 4$$

Thus,

the value of a is 4.

(c) Here,

$$\text{Let, } p(x) = ax^3 + 2x^2 - 3$$

$$f(x) = x^2 - ax + 4$$

$$g(x) = x - 2$$

Now,

$$x - 2 = 0$$

$$\therefore x = 2$$

Using remainder theorem,

$$R_1 = p(2)$$

$$= a \times 2^3 + 2 \times 2^2 - 3$$

$$= 8a + 8 - 3$$

$$= 8a + 5$$

Also,

$$R_2 = f(2)$$

$$= 2^2 - a \times 2 + 4$$

$$= 4 - 2a + 4$$

$$= 8 - 2a$$

We have,

$$p(2) = f(2)$$

$$8a + 5 = 8 - 2a$$

$$\text{or, } 8a + 2a = 8 - 5$$

$$\text{or, } 10a = 3$$

$$\therefore a = \frac{3}{10}$$

(d) Here,
Let,

$$p(x) = px^3 - 3x^2 + 9$$

$$q(x) = x^2 - px + 4$$

$$r(x) = x + 1$$

Now,

$$x + 1 = 0$$

$$\therefore x = -1$$

Using remainder theorem,

$$R_1 = p(-1)$$

$$= p(-1)^3 - 3(-1)^2 + 9$$

$$= p(-1) - 3 + 9$$

$$= -p - 3 + 9$$

$$= 6 - p$$

also,

$$R_2 = q(-1)$$

$$= (-1)^2 - p(-1) + 4$$

$$= 1 + p + 4$$

$$= p + 5$$

We have,

$$p(-1) = q(-1)$$

$$6 - p = p + 5$$

$$\text{or, } 6 - 5 = p + p$$

$$\text{or, } 1 = 2p$$

$$\therefore p = \frac{1}{2}$$

(e) Here,

$$P(x) = 4x^4 - (p+1)x^2 + 8x + 14$$

$$R = 8.5$$

$$Q(x) = 2x + 1$$

Now,

$$2x + 1 = 0$$

$$\therefore x = -\frac{1}{2}$$

Using remainder theorem,

$$R = P\left(-\frac{1}{2}\right)$$

$$\therefore 8.5 = 4 \times \left(-\frac{1}{2}\right)^4 - (p+1) \times \left(-\frac{1}{2}\right)^2 + 8 \times \left(-\frac{1}{2}\right) + 14$$

$$\therefore 8.5 = 4 \times \frac{1}{16} - (p+1) \times \frac{1}{4} - 4 + 14$$

$$\therefore 8.5 = \frac{1}{4} - \frac{p+1}{4} + 10$$

$$\therefore 8.5 = \frac{40 - p - 1 + 40}{4}$$

$$\therefore 34 = 40 - p$$

$$\therefore p = 40 - 34$$

$$\therefore p = 6$$

Q.5. Solⁿ

(a) Here,

$$f(x) = x^3 + ax^2 - x + 4$$

$$g(x) = 2x^3 - x^2 - (a+3)x + 10$$

$$Q(x) = x - 1$$

Now,

$$x - 1 = 0$$

$$\therefore x = 1$$

Using remainder theorem,

$$R_1 = f(1)$$

$$= 1^3 + a \cdot 1^2 - 1 + 4$$

$$= 1 + a + 3$$

$$= a + 4$$

Also,

$$R_2 = g(1)$$

$$= 2 \cdot 1^3 - 1^2 - (a+3) \cdot 1 + 10$$

$$= 2 - 1 - a - 3 + 10$$

$$= 8 - a$$

We have,

$$f(1) = g(1)$$

$$a + 4 = 8 - a$$

$$\therefore a + a = 8 - 4$$

$$\therefore 2a = 4$$

$$\therefore a = 2$$

(b) Here,

$$f(x) = px^3 + 3x^2 - 13$$

$$g(x) = 2x^3 - 5x + p$$

$$Q(x) = x - 2$$

Now,

$$x - 2 = 0$$

$$\therefore x = 2$$

Using remainder theorem,

$$R_1 = f(2)$$

$$= p \times 2^3 + 3 \times 2^2 - 13$$

$$= 8p + 12 - 13$$

$$= 8p - 1$$

Also,

$$R_2 = g(2)$$

$$= 2 \times 2^3 - 5 \times 2 + p$$

$$= 16 - 10 + p$$

$$= p + 6$$

We have,

$$f(2) = g(2)$$

$$8p - 1 = p + 6$$

$$\text{or, } 8p - p = 6 + 1$$

$$\text{or, } 7p = 7$$

$$\therefore p = 1$$

(c) Here,

$$f(x) = x^3 + mx^2 + x - 6$$

$$g(x) = 3x^3 + 2x^2 - (m+1)x + 2$$

$$\phi(x) = x - 1$$

Now,

$$x - 1 = 0$$

$$\therefore x = 1$$

Using remainder theorem,

$$R_1 = f(1)$$

$$= 1^3 + m \times 1^2 + 1 - 6$$

$$= 1 + m - 5$$

$$= m - 4$$

Also,

$$R_2 = g(1)$$

$$= 3 \times 1^3 + 2 \times 1^2 - (m+1) \times 1 + 2$$

$$= 3 + 2 - m - 1 + 2$$

$$= 6 - m$$

We have,

$$f(1) = g(1)$$

$$m - 4 = 6 - m$$

$$\text{or, } m + m = 6 + 4$$

$$\text{or, } 2m = 10$$

$$\therefore m = 5$$

Here,

$$f(x) = 2x^3 - (k-2)x^2 - 7x + 14$$

$$g(x) = 3x^3 + 8x^2 - kx + 6$$

$$d(x) = x + 2$$

Now,

$$x + 2 = 0$$

$$\therefore x = -2$$

Using remainder theorem,

$$R_1 = f(-2)$$

$$= 2 \times (-2)^3 - (k-2) \times (-2)^2 - 7 \times (-2) + 14$$

$$= -16 - (k-2) \times 4 + 14 + 14$$

$$= -16 - 4k + 8 + 28$$

$$= 20 - 4k$$

Also,

$$R_2 = g(-2)$$

$$= 3 \times (-2)^3 + 8 \times (-2)^2 - k \times (-2) + 6$$

$$= -24 + 32 + 2k + 6$$

$$= 2k + 14$$

We have,

$$f(-2) = g(-2)$$

$$\therefore 20 - 4k = 2k + 14$$

$$\therefore 20 - 14 = 2k + 4k$$

$$\therefore 6 = 6k$$

$$\therefore k = 1$$

Q.6 Solⁿ

(a) Here,

$$f(x) = 2x^3 + hx^2 - 10x + 8$$

$$g(x) = 6x^3 - hx^2 + 6$$

$$\phi(x) = 2x - 1$$

Now,

$$2x - 1 = 0$$

$$\therefore x = \frac{1}{2}$$

Using remainder theorem,

$$R_1 = f\left(\frac{1}{2}\right)$$

$$= 2 \times \left(\frac{1}{2}\right)^3 + h \times \left(\frac{1}{2}\right)^2 - 10 \times \frac{1}{2} + 8$$

$$= 2 \times \frac{1}{8} + h \times \frac{1}{4} - 10 \times \frac{1}{2} + 8$$

$$= \frac{1}{4} + \frac{h}{4} + 3$$

$$= \frac{1 + h + 12}{4}$$

$$= \frac{13 + h}{4}$$

Also,

$$R_2 = g\left(\frac{1}{2}\right)$$

$$= 6 \times \left(\frac{1}{2}\right)^3 - h \times \left(\frac{1}{2}\right)^2 + 6$$

$$= 6 \times \frac{1}{8} - h \times \frac{1}{4} + 6$$

$$= \frac{3}{4} - \frac{h}{4} + 6$$

$$= \frac{3 - h + 24}{4}$$

$$= \frac{27 - h}{4}$$

We have,

$$f\left(\frac{1}{2}\right) = g\left(\frac{1}{2}\right)$$

$$\frac{13+h}{4} = \frac{27-h}{4}$$

$$\text{or, } 13+h = 27-h$$

$$\text{or, } h+h = 27-13$$

$$\text{or, } 2h = 14$$

$$\text{or, } h = \frac{14}{2}$$

$$\therefore h = 7$$

(b) Here,

$$f(x) = 3x^3 + 9x^2 - 4x + 2$$

$$g(x) = 9x^3 - 9x^2 + 10$$

$$d(x) = 3x - 1$$

Now,

$$3x - 1 = 0$$

$$\text{or, } 3x = 1$$

$$\therefore x = \frac{1}{3}$$

Using remainder theorem,

$$R_1 = f\left(\frac{1}{3}\right)$$

$$= 3 \times \left(\frac{1}{3}\right)^3 + 9 \times \left(\frac{1}{3}\right)^2 - 4 \times \frac{1}{3} + 2$$

$$= 3 \times \frac{1}{27} + 9 \times \frac{1}{9} - \frac{4}{3} + 2$$

$$= \frac{1}{9} + \frac{9}{9} - \frac{4}{3} + 2$$

$$= \frac{1 + 9 - 4 \times 3 + 2 \times 9}{9}$$

$$= \frac{1 + 9 - 12 + 18}{9}$$

$$= \frac{7 + 9}{9}$$

Also,

$$R_2 = g\left(\frac{1}{3}\right)$$

$$= 9 \times \left(\frac{1}{3}\right)^3 - 9 \times \left(\frac{1}{3}\right)^2 + 10$$

$$= 9 \times \frac{1}{27} - 9 \times \frac{1}{9} + 10$$

$$= \frac{1}{3} - \frac{9}{9} + 10$$

$$= \frac{1 \times 3 - 9 + 9 \times 10}{9}$$

$$= \frac{3 - 9 + 90}{9}$$

$$= \frac{93 - 9}{9}$$

We have,

$$f\left(\frac{1}{3}\right) = g\left(\frac{1}{3}\right)$$

$$\frac{7+9}{9} = \frac{93-9}{9}$$

$$\therefore 7+9 = 93-9$$

$$\therefore 9+9 = 93-7$$

$$\therefore 2 \cdot 9 = 86$$

$$\therefore 9 = 43$$

Q.7 Solⁿ

(a) Here,

Taking 1st case

$$f(x) = ax^2 + bx + 1$$

$$R_1 = 2$$

$$\phi(x) = x - 1$$

Now,

$$x - 1 = 0$$

$$\therefore x = 1$$

Using remainder theorem,

$$R_1 = f(1)$$

$$\text{or, } 2 = a(1)^2 + b(1) + 1$$

$$\text{or, } 2 = a + b + 1$$

$$\text{or, } 2 - 1 = a + b$$

$$\therefore a + b = 1 \longrightarrow (i)$$

Also,

Taking 2nd case

$$f(x) = ax^2 + bx + 1$$

$$R_2 = 4$$

$$\phi(x) = x + 1$$

Now,

$$x + 1 = 0$$

$$\therefore x = -1$$

Using remainder theorem,

$$R_2 = f(-1)$$

$$\text{or, } 4 = a(-1)^2 + b(-1) + 1$$

$$\text{or, } 4 = a - b + 1$$

$$\text{or, } a - b = 4 - 1$$

$$\therefore a - b = 3 \longrightarrow (2)$$

From eqⁿ. (1) and (2), we get

$$a + b = 1$$

$$a - b = 3$$

$$2a = 4$$

$$\therefore a = 2$$

putting the value of a in eqⁿ. (1),

$$2 + b = 1$$

$$\text{or, } b = 1 - 2$$

$$\therefore b = -1$$

Thus,

the values of a and b are 2 and -1 respectively.

(b) Here,

Case - I

$$f(x) = x^2 + bx + c$$

$$g(x) = x + 1$$

$$R_1 = 1$$

Now,

$$x + 1 = 0$$

$$\therefore x = -1$$

Using remainder theorem,

$$R_1 = f(-1)$$

$$\text{or, } 1 = (-1)^2 + b(-1) + c$$

$$\text{or, } 1 = 1 - b + c$$

$$\text{or, } b - c = 1 - 1$$

$$\text{or, } b - c = 0$$

$$\therefore b = c$$

Also,

Case - II

$$f(x) = x^2 + bx + c$$

$$Q(x) = x - 2$$

$$R_2 = 13$$

Using remainder theorem,

$$R_2 = f(2)$$

$$\text{or, } 13 = 2^2 + b \times 2 + c$$

$$\text{or, } 13 = 4 + 2b + c$$

$$\text{or, } 13 - 4 = 2b + c \quad [\because b = c]$$

$$\text{or, } 9 = 3b$$

$$\therefore b = 3$$

Hence, $b = c = 3$ Ans

(c) Here,

Case-I

$$f(x) = x^3 + mx^2 - nx + 6$$

$$R_1 = 24$$

$$Q(x) = x + 2$$

Now,

$$x + 2 = 0$$

$$\therefore x = -2$$

Using remainder theorem,

$$R_1 = f(-2)$$

$$\text{or } 24 = (-2)^3 + m(-2)^2 - n(-2) + 6$$

$$\text{or } 24 = -8 + 4m + 2n + 6$$

$$\text{or } 24 = -2 + 4m + 2n$$

$$\text{or } 24 + 2 = 4m + 2n$$

$$\text{or } 4m + 2n = 26$$

$$\text{or } 2(2m + n) = 26$$

$$\therefore 2m + n = 13 \longrightarrow (1)$$

Also,

Case-II

$$f(x) = x^3 + mx^2 - nx + 6$$

$$R_2 = 54$$

$$Q(x) = x - 3$$

Now,

$$x - 3 = 0$$

$$\therefore x = 3$$

Using remainder theorem,

$$R_2 = f(3)$$

$$\text{or, } 54 = 3^3 + m \times 3^2 - n \times 3 + 6$$

$$\text{or, } 54 = 27 + 9m - 3n + 6$$

$$\text{or, } 54 = 33 + 9m - 3n$$

$$\text{or, } 54 - 33 = 9m - 3n$$

$$\text{or, } 9m - 3n = 21$$

$$\text{or, } 3(3m - n) = 21$$

$$\therefore 3m - n = 7 \longrightarrow (2)$$

From eqn. (1) and (2),

$$2m + n = 13$$

$$+ 3m - n = 7$$

$$5m = 20$$

$$\therefore m = 4$$

Again,

putting the value of m in eqn. (1),

$$2 \times 4 + n = 13$$

$$\text{or, } 8 + n = 13$$

$$\text{or, } n = 13 - 8$$

$$\therefore n = 5$$

Thus,

the values of m and n are 4 and 5 respectively.

(d) Here,

Case-I

$$f(x) = ax^3 + bx^2 + 5x + 2$$

$$R_1 = 5$$

$$D(x) = x - 1$$

Now,

$$x - 1 = 0$$

$$\therefore x = 1$$

Using remainder theorem,

$$R_1 = f(1)$$

$$\text{or, } 5 = a \times 1^3 + b \times 1^2 + 5 \times 1 + 2$$

$$\text{or, } 5 = a + b + 5 + 2$$

$$\text{or, } 5 - 5 - 2 = a + b$$

$$\text{or, } -2 = a + b$$

$$\therefore a + b = -2 \longrightarrow (1)$$

Also,

Case-II

$$f(x) = ax^3 + bx^2 + 5x - 2$$

$$R_2 = -16$$

$$D(x) = x + 2$$

Now,

$$x + 2 = 0$$

$$\therefore x = -2$$

Using remainder theorem,

$$R_2 = f(-2)$$

$$\text{or, } -16 = a \times (-2)^3 + b \times (-2)^2 + 5 \times (-2) - 2$$

$$\text{or, } -16 = -8a + 4b - 10 - 2$$

$$\text{or, } -16 = -8a + 4b - 12$$

$$\text{or, } 8a - 4b = -12 + 16$$

$$\text{or, } 4(2a - b) = 4$$

$$\therefore 2a - b = 1 \rightarrow (2)$$

From eqⁿ (1) and (2),

$$a + b = 2$$

$$2a - b = 1$$

$$3a = 3$$

$$\therefore a = 1$$

putting $a = 1$ in eqⁿ (1), we get

$$1 + b = 2$$

$$\text{or, } b = 2 - 1$$

$$\therefore b = 1$$

Thus,

the values of a and b are 1 and 1 respectively.