

Class :- 10

Subject :- Opt Maths

Unit :- 2 (Trigonometry)

→ 17 Marks

Important formulae

Compound Angles

$$1. \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$$

$$2. \sin(A-B) = \sin A \cdot \cos B - \cos A \sin B$$

$$3. \cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$$

$$4. \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$5. \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$6. \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$7. \cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$8. \cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$9. \tan(A+B+C)$$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

Multiple Angles,

$$1. \sin 2A = 2 \sin A \cos A$$

$$\Rightarrow \frac{2 \tan A}{1 + \tan^2 A}$$

$$2. \cos 2A = \cos^2 A - \sin^2 A$$

$$\Rightarrow 2 \cos^2 A - 1$$

$$\Rightarrow 1 - 2 \sin^2 A$$

$$\Rightarrow \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$3. \tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$4. \sin 3A = 3 \sin A - 4 \sin^3 A$$

$$5. \cos 3A = 4 \cos^3 A - 3 \cos A$$

$$6. \tan 3A = \frac{3 \tan A - \tan^3 A}{1 - 3 \tan^2 A}$$

Sub-Multiple Angles

$$1. \sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$$

$$\Rightarrow \frac{2 \tan \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$2. \cos A = \cos^2 \frac{A}{2} - \sin^2 \frac{A}{2}$$

$$\Rightarrow 2 \cos^2 \frac{A}{2} - 1$$

$$\Rightarrow 1 - 2 \sin^2 \frac{A}{2}$$

$$\Rightarrow \frac{1 - \tan^2 \frac{A}{2}}{1 + \tan^2 \frac{A}{2}}$$

$$3. \tan A = \frac{2 \tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}}$$

$$4. \sin A = 3 \sin \frac{A}{3} - 4 \sin^3 \frac{A}{3}$$

$$5. \cos A = \frac{4\cos^3 A}{3} - \frac{3\cos A}{3}$$

$$6. \tan A = \frac{3\tan \frac{A}{3} - \tan^3 \frac{A}{3}}{1 - 3\tan^2 \frac{A}{3}}$$

Transformation of Trigonometric Identities

$$1(a) 2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$(b) 2\cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$(c) 2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$(d) 2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$2(a) \sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$(b) \sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$$

$$(c) \cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$(d) \cos C - \cos D = 2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$$

$$\text{OR } \Rightarrow -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$$

Values of Trigonometric Ratios

Angles	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
cosec	∞	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{\sqrt{3}}$	-1
cot	∞	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	∞

Height & Distance

1. **Angle of Elevation**:- The angle made by upward line of sight with horizontal line is called angle of elevation.

2. **Angle of depression**:- The angle made by downward line of sight with horizontal line is called angle of depression.

Exercise-7.1,

Date

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Q.1 (a) Solⁿ

Here,

$$5 \sin A \cos B = 5 \cos A \sin B + 3$$

$$\text{or, } 5 \sin A \cos B - 5 \cos A \sin B = 3$$

$$\text{or, } 5 (\sin A \cos B - \cos A \sin B) = 3$$

$$\text{or, } \sin A \cos B - \cos A \sin B = \frac{3}{5}$$

$$\therefore \sin(A-B) = \frac{3}{5} \text{ Ans}$$

(b) Here,

$$5 \cos A \cos B = 4 + 5 \sin A \sin B$$

$$\text{or, } 5 \cos A \cos B - 5 \sin A \sin B = 4$$

$$\text{or, } 5 (\cos A \cos B - \sin A \sin B) = 4$$

$$\text{or, } \cos A \cos B - \sin A \sin B = \frac{4}{5}$$

$$\therefore \cos(A+B) = \frac{4}{5} \text{ Ans}$$

Q.2 (a) Solⁿ

Here,

$$\tan A + \tan B = 4(1 - \tan A \tan B)$$

$$\text{or, } \frac{\tan A + \tan B}{1 - \tan A \tan B} = 4$$

$$\therefore \tan(A+B) = 4$$

(b) Solⁿ

Here,

$$5(\cot A + \cot B) = \cot A \cot B - 1$$

$$\text{or, } 5 = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\text{or, } 5 = \cot(A+B)$$

$$\therefore \cot(A+B) = 5$$

(c) Solⁿ

Here,

$$\tan A - \tan B = 8(1 + \tan A \tan B)$$

$$\text{or, } \frac{\tan A - \tan B}{1 + \tan A \tan B} = 8$$

$$\therefore \tan(A-B) = 8$$

(d) Here

$$10(\cot B - \cot A) = \cot A \cot B + 1$$

$$\text{or, } 10 = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

$$\text{or, } 10 = \cot(A - B)$$

$$\therefore \cot(A - B) = 10$$

Q. 3. (a) Soln

Here,

$$2 \cos(A + B) = \sqrt{3}$$

$$\text{or, } \cos(A + B) = \frac{\sqrt{3}}{2}$$

$$\text{or, } \cos(A + B) = \cos 30^\circ$$

$$\therefore A + B = 30^\circ$$

(b) Here,

$$2 \sin(A+B) = 1$$

$$\text{or, } \sin(A+B) = \frac{1}{2}$$

$$\text{or, } \sin(A+B) = \sin 30^\circ$$

$$\therefore A+B = 30^\circ$$

(c) Here,

$$\sqrt{3} \tan(A+B) = 1$$

$$\text{or, } \tan(A+B) = \frac{1}{\sqrt{3}}$$

$$\text{or, } \tan(A+B) = \tan 30^\circ$$

$$\therefore A+B = 30^\circ$$

(d) Here,

$$3 \cot(A-B) = \sqrt{3}$$

$$\text{or, } \cot(A-B) = \frac{\sqrt{3}}{3}$$

$$\text{or, } \cot(A-B) = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$\text{or, } \cot(A-B) = \frac{1}{\sqrt{3}}$$

$$\text{or, } \cot(A-B) = \cot 60^\circ$$

$$\therefore A-B = 60^\circ$$

Q. 4(a) Solⁿ

Here,

$$\begin{aligned} \sin 75^\circ &= \sin(45^\circ + 30^\circ) \\ &= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} \end{aligned}$$

(b) Here,

$$\begin{aligned} \cos 75^\circ &= \cos(45^\circ + 30^\circ) \\ &= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \\ &= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} \end{aligned}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Here,

(c) $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= 2 + \sqrt{3}$$

(d) Here,

$$\sin 15^\circ = \sin(45^\circ - 30^\circ)$$

$$= \sin 45^\circ \cdot \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

(e) Here

$$\tan 15^\circ = \tan (60^\circ - 45^\circ)$$

$$= \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \cdot \tan 45^\circ}$$

$$= \frac{\sqrt{3} - 1}{1 + \sqrt{3} \cdot 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{3 - \sqrt{3} - \sqrt{3} + 1}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{4 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{2(2 - \sqrt{3})}{2}$$

$$= 2 - \sqrt{3} \text{ Ans}$$

Here,

$$(f) \tan 105^\circ = \tan(60^\circ + 45^\circ)$$

$$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \cdot \tan 45^\circ}$$

$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1}$$

$$= \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}}$$

$$= \frac{1 + \sqrt{3} + \sqrt{3} + 3}{1^2 - (\sqrt{3})^2}$$

$$= \frac{4 + 2\sqrt{3}}{1 - 3}$$

$$= \frac{2(2 + \sqrt{3})}{-2}$$

$$= -(2 + \sqrt{3})$$

$$= -2 - \sqrt{3}$$

(g) Here,

$$\cos 105^\circ = \cos(60^\circ + 45^\circ)$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

(h) Here,

$$\tan 195^\circ = \tan(150^\circ + 45^\circ)$$

$$= \frac{\tan 150^\circ + \tan 45^\circ}{1 - \tan 150^\circ \cdot \tan 45^\circ}$$

$$= \frac{-\frac{1}{\sqrt{3}} + 1}{1 - \left(-\frac{1}{\sqrt{3}}\right) \cdot 1}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{3 - \sqrt{3} - \sqrt{3} + 1}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{4 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{2(2 - \sqrt{3})}{2}$$

$$= 2 - \sqrt{3}$$



Q.5. Soln.

(a) Here,

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

L.H.S.

$$\cos 15^\circ = \cos (45^\circ - 30^\circ)$$

$$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} = \text{R.H.S}$$

∴ Hence, proved.

(b) Here,

$$\sin 105^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

L.H.S.

$$\sin 105^\circ = \sin (60^\circ + 45^\circ)$$

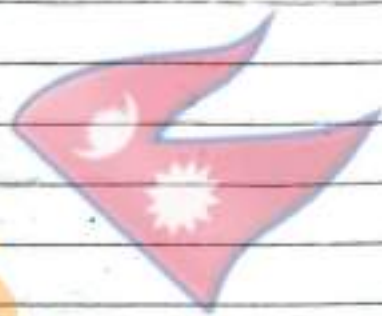
$$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} = \text{RHS.}$$

∴ Hence, proved.



(c) Here,

$$\cos 195^\circ = -\frac{\sqrt{3}+1}{2\sqrt{2}}$$

L.H.S.

$$\cos 195^\circ = \cos(150^\circ + 45^\circ)$$

$$= \cos 150^\circ \cos 45^\circ - \sin 150^\circ \sin 45^\circ$$

$$= -\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= -\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$$

$$= -\frac{\sqrt{3}+1}{2\sqrt{2}} = \text{RHS}$$

Hence, proved.

(d) Here,

$$\cot 15^\circ = 2 + \sqrt{3}$$

L.H.S.

$$\cot 15^\circ = \cot (45^\circ - 30^\circ)$$

$$= \frac{\cot 45^\circ \cot 30^\circ + 1}{\cot 30^\circ - \cot 45^\circ}$$

$$= \frac{1 \cdot \sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$$

$$= \frac{3 + \sqrt{3} + \sqrt{3} + 1}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{4 + 2\sqrt{3}}{3 - 1}$$

$$= \frac{2(2 + \sqrt{3})}{2}$$

$$= 2 + \sqrt{3} = \text{RHS}$$

Hence, proved.

Here,

$$(e) \cot 75^\circ = 2 - \sqrt{3}$$

L.H.S.

$$\cot 75^\circ = \cot (45^\circ + 30^\circ)$$

$$= \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot 30^\circ + \cot 45^\circ}$$

$$= \frac{1 \cdot \sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{3 - \sqrt{3} - \sqrt{3} + 1}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{4 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{2(2 - \sqrt{3})}{2}$$

$$= 2 - \sqrt{3} = \text{RHS}$$

Hence, proved

Here,

$$(7) \cot 105^\circ = \sqrt{3} - 2$$

L.H.S.

$$\cot 105^\circ = \cot (60^\circ + 45^\circ)$$

$$= \frac{\cot 60^\circ \cot 45^\circ - 1}{\cot 45^\circ + \cot 60^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}} \cdot 1 - 1}{1 + \frac{1}{\sqrt{3}}}$$

$$= \frac{1 - \sqrt{3}}{\sqrt{3} + 1}$$

$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$

$$= \frac{1 - \sqrt{3} - \sqrt{3} + 3}{1^2 - (\sqrt{3})^2}$$

$$= \frac{4 - 2\sqrt{3}}{1 - 3}$$

$$= \frac{2(2 - \sqrt{3})}{-2}$$

$$= \sqrt{3} - 2$$

Q.6 Solⁿ

Here,

$$\sin A = \frac{4}{5}$$

$$\Rightarrow \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\sin B = \frac{3}{7}$$

$$\Rightarrow \cos B = \sqrt{1 - \sin^2 B} = \sqrt{1 - \left(\frac{3}{7}\right)^2} = \frac{2\sqrt{10}}{7}$$

(a) Here,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5} \times \frac{2\sqrt{10}}{7} + \frac{3}{5} \times \frac{3}{7}$$

$$= \frac{8\sqrt{10}}{35} + \frac{9}{35}$$

$$= \frac{8\sqrt{10} + 9}{35}$$

(b) Here,

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{4}{5} \times \frac{2\sqrt{10}}{7} - \frac{3}{5} \times \frac{3}{7}$$

$$= \frac{8\sqrt{10}}{35} - \frac{9}{35}$$

$$= \frac{8\sqrt{10} - 9}{35}$$

(c) Here,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{3}{5} \times \frac{2\sqrt{10}}{7} - \frac{4}{5} \times \frac{3}{7}$$

$$= \frac{6\sqrt{10}}{35} - \frac{12}{35}$$

$$= \frac{6\sqrt{10} - 12}{35}$$

(d) Here,

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \times \frac{2\sqrt{10}}{7} + \frac{4}{5} \times \frac{3}{7}$$

$$= \frac{6\sqrt{10}}{35} + \frac{12}{35}$$

$$= \frac{6\sqrt{10} + 12}{35}$$

$$(e) \tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{\frac{4}{5} \times \frac{2\sqrt{10}}{7} + \frac{3}{5} \times \frac{3}{7}}{\frac{3}{5} \times \frac{2\sqrt{10}}{7} - \frac{4}{5} \times \frac{3}{7}}$$

$$= \frac{\frac{8\sqrt{10}}{35} + \frac{9}{35}}{\frac{6\sqrt{10}}{35} - \frac{12}{35}}$$

$$= \frac{8\sqrt{10} + 9}{6\sqrt{10} - 12}$$

$$= \frac{8\sqrt{10} + 9}{6\sqrt{10} - 12}$$

$$= \frac{8\sqrt{10} + 9}{35} - \frac{6\sqrt{10} - 12}{35}$$

$$= \frac{8\sqrt{10} + 9}{6\sqrt{10} - 12}$$

Here,

$$(f) \tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)}$$

$$= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{\frac{4}{5} \times \frac{2\sqrt{10}}{7} - \frac{3}{5} \times \frac{3}{7}}{\frac{3}{5} \times \frac{2\sqrt{10}}{7} + \frac{4}{5} \times \frac{3}{7}}$$

$$= \frac{\frac{8\sqrt{10}}{35} - \frac{9}{35}}{\frac{6\sqrt{10}}{35} + \frac{12}{35}}$$

$$= \frac{8\sqrt{10} - 9}{6\sqrt{10} + 12} = \frac{8\sqrt{10} - 9}{6\sqrt{10} + 12}$$

Q7 Solⁿ

Here,

$$\cos A = \frac{4}{5}$$

$$\sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

$$\cos B = \frac{12}{13}$$

$$\sin B = \sqrt{1 - \cos^2 B} = \sqrt{1 - \left(\frac{12}{13}\right)^2} = \frac{5}{13}$$

(a) Here,

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{3}{5} \times \frac{12}{13} + \frac{4}{5} \times \frac{5}{13}$$

$$= \frac{36}{65} + \frac{20}{65}$$

$$= \frac{36 + 20}{65}$$

$$= \frac{56}{65}$$

Here,

$$(b) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13}$$

$$= \frac{48}{65} - \frac{15}{65}$$

$$= \frac{48-15}{65}$$

$$= \frac{33}{65}$$

(c) Here,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

Now,

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{3}{4} \times \frac{5}{12}}$$

$$= \frac{\frac{3}{4} - \frac{5}{12}}{1 + \frac{3}{4} \times \frac{5}{12}}$$

$$\begin{aligned} &= \frac{3 \times 3 - 5 \times 1}{12} \\ &= \frac{48 + 15}{48} \\ &= \frac{4}{12} \times \frac{48}{63} \\ &= \frac{16}{63} \end{aligned}$$

(d) Here,

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{3}{5} \times \frac{12}{13} - \frac{4}{5} \times \frac{5}{13}$$

$$= \frac{36}{65} - \frac{20}{65}$$

$$= \frac{36 - 20}{65}$$

$$= \frac{16}{65}$$

$$(c) \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{4}{5} \times \frac{12}{13} + \frac{3}{5} \times \frac{5}{13}$$

$$= \frac{48}{65} + \frac{15}{65}$$

$$= \frac{48+15}{65}$$

$$= \frac{63}{65}$$

$$(f) \text{ Here, } \tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\tan B = \frac{\sin B}{\cos B} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

$$\text{Now, } \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{3}{4} + \frac{5}{12}$$

$$= 1 - \frac{3}{4} \times \frac{5}{12}$$

$$= \frac{3 \times 3 + 5 \times 1}{12}$$

$$= \frac{48 - 15}{48}$$

$$= \frac{14}{12} \times \frac{48}{33}$$

$$= \frac{56}{33}$$



8. Soln

(a) Here,

$$\sin(A+B) + \sin(A-B)$$

$$= (\sin A \cos B + \cos A \sin B) + (\sin A \cos B - \cos A \sin B)$$

$$= \sin A \cos B + \cancel{\cos A \sin B} + \sin A \cos B - \cancel{\cos A \sin B}$$

$$= 2 \sin A \cos B$$

(b) Here,

$$\sin(A+B) - \sin(A-B)$$

$$= (\sin A \cos B + \cos A \sin B) - (\sin A \cos B - \cos A \sin B)$$

$$= \sin A \cancel{\cos B} + \cos A \sin B - \sin A \cancel{\cos B} + \cos A \sin B$$

$$= 2 \cos A \sin B$$

(c) Here,

$$\cos(A+B) + \cos(A-B)$$

$$= (\cos A \cos B - \sin A \sin B) + (\cos A \cos B + \sin A \sin B)$$

$$= \cos A \cos B - \cancel{\sin A \sin B} + \cos A \cos B + \cancel{\sin A \sin B}$$

$$= 2 \cos A \cos B$$

(d) Here,

$$\cos(A+B) - \cos(A-B)$$

$$= (\cos A \cos B - \sin A \sin B) - (\cos A \cos B + \sin A \sin B)$$

$$= \cos A \cos B - \sin A \sin B - \cos A \cos B - \sin A \sin B$$

$$= -2 \sin A \sin B$$

Q. 9. Solⁿ

(a) Here,

$$\sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$$

L.H.S.

$$\sin(A+B) \cdot \sin(A-B)$$

$$= (\sin A \cos B + \cos A \sin B) \cdot (\sin A \cos B - \cos A \sin B)$$

$$= \sin^2 A \cos^2 B - \sin A \cos B \cos A \sin B + \sin A \cos B \cos A \sin B - \cos^2 A \sin^2 B$$

$$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$$

$$= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B$$

$$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$$

$$= \sin^2 A - \sin^2 B = \text{RHS}$$

$\therefore$ Hence, proved.

(b) Here,

$$\cos(A+B) \cdot \cos(A-B) = \cos^2 B - \sin^2 A$$

$$= (\cos A \cos B - \sin A \sin B)(\cos A \cos B + \sin A \sin B)$$

$$= \cos^2 A \cos^2 B - \sin^2 A \sin^2 B$$

$$= \cos^2 B (1 - \sin^2 A) - \sin^2 A (1 - \cos^2 B)$$

$$= \cos^2 B - \sin^2 A / \cos^2 B - \sin^2 A + \sin^2 A / \cos^2 B$$

$$= \cos^2 B - \sin^2 A = \text{RHS}$$

Hence, proved.

(c) Here,

$$\tan(A+B) + \tan(A-B) = \frac{\sin 2A}{\cos^2 A - \sin^2 B}$$

L.H.S.

$$\tan(A+B) + \tan(A-B)$$

$$= \frac{\sin(A+B)}{\cos(A+B)} + \frac{\sin(A-B)}{\cos(A-B)}$$

$$= \frac{\sin(A+B)\cos(A-B) + \cos(A+B)\sin(A-B)}{\cos(A+B)\cos(A-B)}$$

$$= \frac{\sin(A+B+A-B)}{\cos^2 A - \sin^2 B}$$

$$= \frac{\sin 2A}{\cos^2 A - \sin^2 B}$$

(d) Here,

$$\tan(A+B) \cdot \tan(A-B) = \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$$

L.H.S.

$$\tan(A+B) \cdot \tan(A-B)$$

$$= \frac{\sin(A+B)}{\cos(A+B)} \cdot \frac{\sin(A-B)}{\cos(A-B)}$$

$$= \frac{\sin(A+B) \cdot \sin(A-B)}{\cos(A+B) \cdot \cos(A-B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B} = \text{RHS.}$$

Hence, proved.

(e) Here,

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

L.H.S.

$$\cot(A+B)$$

$$= \frac{\cos(A+B)}{\sin(A+B)}$$

$$= \frac{\cos A \cos B - \sin A \sin B}{\sin A \cos B + \cos A \sin B}$$

$$\begin{aligned}
 &= \frac{\cos A \cdot \cos B}{\sin A \cdot \sin B} - \frac{\sin A \cdot \sin B}{\sin A \cdot \sin B} \\
 &= \frac{\sin A \cdot \cos B}{\sin A \cdot \sin B} + \frac{\cos A \cdot \sin B}{\sin A \cdot \sin B} \\
 &= \frac{\cot A \cot B - 1}{\cot B + \cot A} = \text{RHS.}
 \end{aligned}$$

Hence, proved.

(f) Here,

$$\cot(A-B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

L.H.S.

$$\cot(A-B) = \frac{\cos(A-B)}{\sin(A-B)}$$

$$= \frac{\cos A \cdot \cos B + \sin A \sin B}{\sin A \cos B - \cos A \sin B}$$

$$\begin{aligned}
 &= \frac{\cos A \cos B}{\sin A \sin B} + \frac{\sin A \sin B}{\sin A \sin B} \\
 &\quad - \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B}
 \end{aligned}$$

$$= \frac{\cot A \cot B + 1}{\cot B + \cot A} = \text{RHS}$$

Q.10. Soln.

(a) Here,

$$\frac{\cos(A+B)}{\sin A \cos B} = \cot A - \tan B$$

L.H.S.

$$\frac{\cos(A+B)}{\sin A \cdot \cos B}$$

$$= \frac{\cos A \cos B - \sin A \sin B}{\sin A \cos B}$$

$$= \frac{\cos A \cos B}{\sin A \cos B} - \frac{\sin A \sin B}{\sin A \cos B}$$

$$= \cot A - \tan B = \text{RHS}$$

Hence, proved.



(b) Here,

$$\frac{\sin(A-B)}{\sin A \cos B} = 1 - \cot A \tan B$$

L.H.S.

$$\frac{\sin(A-B)}{\sin A \cos B}$$

$$= \frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B}$$

$$= \frac{\sin A \cos B}{\sin A \cos B} - \frac{\cos A \sin B}{\sin A \cos B}$$

$$= 1 - \cot A \tan B = \text{RHS}$$

Hence, proved.

(c) Here,

$$\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan A + \tan B}{\tan A - \tan B}$$

L.H.S.

$$\frac{\sin(A+B)}{\sin(A-B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B}$$

$$= \frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}$$

$$= \frac{\sin A \cos B}{\cos A \cos B} + \frac{\cos A \sin B}{\cos A \cos B}$$

$$= \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}$$

$$\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}$$

$$= \frac{\tan A + \tan B}{\tan A - \tan B} = \text{R.H.S.}$$

Hence, proved.

(d) Here,

$$\sec(A+B) = \frac{\sec A \sec B}{1 - \tan A \tan B}$$

L.H.S.

$$\sec(A+B) = \frac{1}{\cos(A+B)}$$

$$= \frac{1}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{1}{\cos A \cos B} \cdot \frac{\cos A \cos B}{\cos A \cos B - \sin A \sin B}$$

$$= \frac{\sec A \sec B}{1 - \tan A \tan B} = \text{R.H.S.}$$

Hence, proved.

(e) Here,
 $1 + \tan 2A \cdot \tan A = \sec 2A$

L.H.S.

$$1 + \tan 2A \tan A$$

$$= 1 + \frac{\sin 2A}{\cos 2A} \cdot \frac{\sin A}{\cos A}$$

$$= \frac{\cos 2A \cos A + \sin 2A \sin A}{\cos 2A \cdot \cos A}$$

$$= \frac{\cos(2A - A)}{\cos 2A \cdot \cos A}$$

$$= \frac{\cos A^1}{\cos 2A \cdot \cos A}$$

$$= \sec 2A = R.H.S.$$

Hence, proved.

(f) Here,
 $\cot 2A + \tan A = \csc 2A$
L.H.S.
 $\cot 2A + \tan A$

$$= \frac{\cos 2A}{\sin 2A} + \frac{\sin A}{\cos A}$$

$$= \frac{\cos 2A \cos A + \sin 2A \sin A}{\sin 2A \cdot \cos A}$$

$$= \frac{\cos(2A - A)}{\sin 2A \cdot \cos A}$$

$$= \frac{\cos A}{\sin 2A \cdot \cos A}$$

$$= \frac{1}{\sin 2A}$$

$$= \csc 2A = \text{R.H.S.}$$

Hence, proved.

Q.11. Soln

$$(a) \cos(120^\circ + A) + \cos(120^\circ - A) = -\cos A$$

L.H.S.

$$\cos(120^\circ + A) + \cos(120^\circ - A)$$

$$= \cos 120^\circ \cdot \cos A - \sin 120^\circ \cdot \sin A + \cos 120^\circ \cdot \cos A + \sin 120^\circ \cdot \sin A$$

$$= -\frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A - \frac{1}{2} \cos A + \frac{\sqrt{3}}{2} \sin A$$

$$= -2 \times \frac{1}{2} \cos A$$

$$= -\cos A = \text{R.H.S.}$$

Hence, proved.

(b) Here

$$\tan(45^\circ + A) = \frac{\cos A + \sin A}{\cos A - \sin A}$$

L.H.S.

$$\tan(45^\circ + A)$$

$$= \frac{\sin(45^\circ + A)}{\cos(45^\circ + A)}$$

$$= \frac{\sin 45^\circ \cos A + \cos 45^\circ \sin A}{\cos 45^\circ \cos A - \sin 45^\circ \sin A}$$

$$= \frac{\frac{1}{\sqrt{2}} \cos A + \frac{1}{\sqrt{2}} \sin A}{\frac{1}{\sqrt{2}} \cos A - \frac{1}{\sqrt{2}} \sin A}$$

$$= \frac{\frac{1}{\sqrt{2}} (\cos A + \sin A)}{\frac{1}{\sqrt{2}} (\cos A - \sin A)}$$

$$= \frac{\cos A + \sin A}{\cos A - \sin A} = \text{RHS.}$$

Hence, proved.

(c) Here,
 $\tan(45^\circ + A) \cdot \tan(45^\circ - A) = 1$

L.H.S.

$$\tan(45^\circ + A) \cdot \tan(45^\circ - A)$$

$$= \frac{\tan 45^\circ + \tan A}{1 - \tan 45^\circ \tan A} \times \frac{\tan 45^\circ - \tan A}{1 + \tan 45^\circ \tan A}$$

$$= \frac{1 + \tan A}{1 - \tan A} \times \frac{1 - \tan A}{1 + \tan A}$$

$$= 1 = \text{RHS.}$$

Hence, proved.

(d) Here,
 $\cot(45^\circ + A) = \frac{\cot A - 1}{\cot A + 1}$

L.H.S.

$$\cot(45^\circ + A) = \frac{\cot 45^\circ \cot A - 1}{\cot A + \cot 45^\circ}$$

$$= \frac{1 \cdot \cot A - 1}{\cot A + 1}$$

$$= \frac{\cot A - 1}{\cot A + 1} = \text{R.H.S.}$$

(e) Here,

$$\cos(60^\circ - A) \cdot \cos(30^\circ - B) - \sin(60^\circ - A) \cdot \sin(30^\circ - B) = \sin(A + B)$$

L.H.S.

$$\cos(60^\circ - A) \cos(30^\circ - B) - \sin(60^\circ - A) \cdot \sin(30^\circ - B)$$

$$= \cos[(60^\circ - A) + (30^\circ - B)]$$

$$= \cos(60^\circ - A + 30^\circ - B)$$

$$= \cos[90^\circ - (A + B)]$$

$$= \sin(A + B) = \text{R.H.S.}$$

Hence, proved.

(f) Here,

$$\sin(45^\circ + A) \cos(45^\circ - B) + \cos(45^\circ + A) \cdot \sin(45^\circ - B) = \cos(A - B)$$

L.H.S.

$$\sin(45^\circ + A) \cdot \cos(45^\circ - B) + \cos(45^\circ + A) \cdot \sin(45^\circ - B)$$

$$= \sin[(45^\circ + A) + (45^\circ - B)]$$

$$= \sin(90^\circ + (A - B))$$

$$= \cos(A - B) = \text{R.H.S.}$$

Hence, proved.

Q-12 Solⁿ

(a) Here,

$$\sin 38^\circ \cos 22^\circ + \cos 38^\circ \sin 22^\circ$$

$$= \sin(38^\circ + 22^\circ)$$

$$= \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2}$$

(b) Here,

$$\cos 70^\circ \cos 10^\circ + \sin 70^\circ \sin 10^\circ$$

$$= \cos(70^\circ - 10^\circ)$$

$$= \cos 60^\circ$$

$$= \frac{1}{2}$$

(c) Here,

$$\sin(x-y) \cos x - \cos(x-y) \sin x$$

$$= \sin(x-y-x)$$

$$= \sin(-y)$$

$$= -\sin y$$

(d) Here,

$$\frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ}$$

$$= \tan (69^\circ + 66^\circ)$$

$$= \tan 135^\circ$$

$$= -1$$

Q. 13. Solⁿ.

(a) Here,

$$\cos 105^\circ + \sin 105^\circ$$

$$= \cos (60^\circ + 45^\circ) + \sin (60^\circ + 45^\circ)$$

$$= (\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ) + (\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ)$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

(b) Here,

$$\cos 15^\circ - \sin 15^\circ$$

$$= \cos(45^\circ - 30^\circ) - \sin(45^\circ - 30^\circ)$$

$$= (\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ) - (\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right) - \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

(c) Here,

$$\cot 105^\circ - \tan 105^\circ$$

$$= \cot(60^\circ + 45^\circ) - \tan(60^\circ + 45^\circ)$$

$$= \frac{\cot 60^\circ \cdot \cot 45^\circ - 1}{\cot 45^\circ + \cot 60^\circ} - \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \cdot \tan 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{3}} \times 1 - 1}{1 + \frac{1}{\sqrt{3}}} - \frac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1}$$

$$= \frac{\frac{1 - \sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} - \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$$

$$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} - \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

$$= \frac{(1 - \sqrt{3})^2 - (1 + \sqrt{3})^2}{(1 + \sqrt{3})(1 - \sqrt{3})}$$

$$= \frac{\{1^2 - 2 \times 1 \times \sqrt{3} + (\sqrt{3})^2\} - \{1^2 + 2 \times 1 \times \sqrt{3} + (\sqrt{3})^2\}}{1^2 - (\sqrt{3})^2}$$

$$= \frac{1 - 2\sqrt{3} + 3 - 1 - 2\sqrt{3} - 3}{1 - 3}$$

$$= \frac{-4\sqrt{3}}{-2}$$

$$= +2\sqrt{3}$$

(d) Here,

$$\tan 15^\circ + \cot 15^\circ$$

$$= \tan(45^\circ - 30^\circ) + \cot(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} + \frac{\cot 45^\circ \cot 30^\circ + 1}{\cot 30^\circ - \cot 45^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} + \frac{1 \cdot \sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} + \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} + \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{(\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2}{(\sqrt{3} + 1)(\sqrt{3} - 1)}$$

$$= \frac{\{(\sqrt{3})^2 - 2 \times \sqrt{3} \times 1 + 1^2\} + \{(\sqrt{3})^2 + 2 \times \sqrt{3} \times 1 + 1^2\}}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{3 - 2\sqrt{3} + 1 + 3 + 2\sqrt{3} + 1}{3 - 1}$$

$$= \frac{8}{2}$$

$$= 4$$

(e) Here,

$$\cos 105^\circ - \sin 15^\circ$$

$$= \cos(60^\circ + 45^\circ) - \sin(60^\circ - 45^\circ)$$

$$= (\cos 60^\circ \cdot \cos 45^\circ - \sin 60^\circ \sin 45^\circ) - (\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ)$$

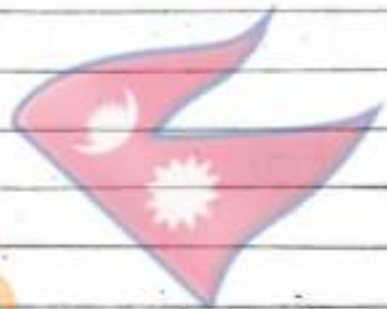
$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}} - \frac{2\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \frac{1 - \sqrt{3}}{\sqrt{2}}$$



(f) Here,

$$\sin 165^\circ - \cos 165^\circ$$

$$= \sin(120^\circ + 45^\circ) - \cos(120^\circ + 45^\circ)$$

$$= (\sin 120^\circ \cos 45^\circ + \cos 120^\circ \sin 45^\circ) - (\cos 120^\circ \cos 45^\circ - \sin 120^\circ \sin 45^\circ)$$

$$= \left\{ \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \left(-\frac{1}{2}\right) \times \frac{1}{\sqrt{2}} \right\} - \left\{ \left(-\frac{1}{2}\right) \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \right\}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{2\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \sqrt{\frac{3}{2}}$$

Q.14. Soln.

(a) Here,

$$\sin 75^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$$

L.H.S.

$$\sin 75^\circ - \sin 15^\circ$$

$$= \sin(45^\circ + 30^\circ) - \sin(45^\circ - 30^\circ)$$

$$= (\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ) - (\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right) - \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} = \text{R.H.S.}$$

Hence, proved.

(b) Here,

$$\cos 15^\circ - \cos 75^\circ = \frac{1}{\sqrt{2}}$$

L.H.S.

$$\cos 15^\circ - \cos 75^\circ$$

$$= \cos (45^\circ - 30^\circ) - \cos (45^\circ + 30^\circ)$$

$$= (\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ) - (\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ)$$

$$= \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right) - \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} = \text{RHS.}$$

Hence, proved.

(c) Here,

$$\sin 105^\circ + \sin 15^\circ = \frac{\sqrt{3}}{2}$$

= L.H.S.

$$\sin 105^\circ + \sin 15^\circ$$

$$= \sin(60^\circ + 45^\circ) + \sin(60^\circ - 45^\circ)$$

$$= (\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ) + (\sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ)$$

$$= \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}} \right) + \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{2\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \sqrt{\frac{3}{2}} = \text{RHS.}$$

Hence, proved.

(d) Here,

$$\sin 105^\circ - \cos 75^\circ = \frac{1}{\sqrt{2}}$$

L.H.S.

$$\sin 105^\circ - \cos 75^\circ$$

$$= \sin(60^\circ + 45^\circ) - \cos(45^\circ + 30^\circ)$$

$$= (\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ) - (\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ)$$

$$= \left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}} \right) - \left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} = \text{R.H.S.}$$

Hence, proved.

(e) Here,

$$\cos 105^\circ + \cos 15^\circ = \frac{1}{\sqrt{2}}$$

L.H.S.

$$\cos 105^\circ + \cos 15^\circ$$

$$= \cos(60^\circ + 45^\circ) + \cos(60^\circ - 45^\circ)$$

$$= (\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ) + (\cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ)$$

$$= \left(\frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} \right)$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}} = \text{R.H.S.}$$

Hence, proved.

(f) Here,
 $\tan 15^\circ + \cot 15^\circ = 4$
L.H.S.
 $\tan 15^\circ + \cot 15^\circ$

$$= \frac{\sin 15^\circ}{\cos 15^\circ} + \frac{\cos 15^\circ}{\sin 15^\circ}$$

$$= \frac{\sin 15^\circ \cdot \sin 15^\circ + \cos 15^\circ \cdot \cos 15^\circ}{\sin 15^\circ \cdot \cos 15^\circ}$$

$$= \frac{\sin^2 15^\circ + \cos^2 15^\circ}{\sin 15^\circ \cdot \cos 15^\circ}$$

$$= \frac{2 \times 1}{2 \sin 15^\circ \cdot \cos 15^\circ}$$

$$= \frac{2}{\sin 2 \times 15^\circ}$$

$$= \frac{2}{\sin 30^\circ}$$

$$= \frac{2}{\frac{1}{2}}$$

$$= 4 = \text{R.H.S.}$$



Alternative Method

L.H.S.

$$\tan 15^\circ + \cot 15^\circ$$

$$= \tan(45^\circ - 30^\circ) + \cot(45^\circ - 30^\circ)$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} + \frac{\cot 45^\circ \cot 30^\circ + 1}{\cot 30^\circ - \cot 45^\circ}$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} + \frac{1 \times \sqrt{3} + 1}{\sqrt{3} - 1}$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} + \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2 + (\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{4 - 2\sqrt{3} + 4 + 2\sqrt{3}}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{8}{3-1}$$

$$= \frac{8}{2}$$

$$= 4 = \text{RHS}$$

Hence, proved.

(g) Here,
 $\tan 75^\circ + \cot 75^\circ = 4$

L.H.S.

$$\tan 75^\circ + \cot 75^\circ$$

$$= \tan(45^\circ + 30^\circ) + \cot(45^\circ + 30^\circ)$$

$$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} + \frac{\cot 45^\circ \cot 30^\circ - 1}{\cot 30^\circ + \cot 45^\circ}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \times \frac{1}{\sqrt{3}}} + \frac{1 \times \sqrt{3} - 1}{\sqrt{3} + 1}$$

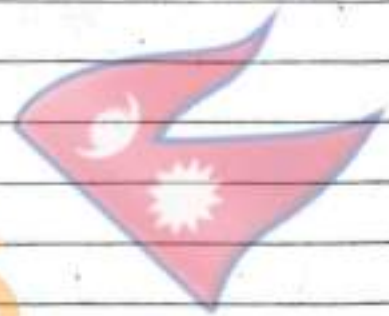
$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1} + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$$

$$= \frac{4+2\sqrt{3}+4-2\sqrt{3}}{3-1}$$

$$= \frac{8}{2}$$

$$= 4$$



(h) Here,

$$\cos 18^\circ - \sin 18^\circ = \sqrt{2} \sin 27^\circ$$

L.H.S.

$$\cos 18^\circ - \sin 18^\circ$$

$$= \cos(45^\circ - 27^\circ) - \sin(45^\circ - 27^\circ)$$

$$= (\cos 45^\circ \cos 27^\circ + \sin 45^\circ \sin 27^\circ) - (\sin 45^\circ \cos 27^\circ - \cos 45^\circ \sin 27^\circ)$$

$$= \frac{1}{\sqrt{2}} \cancel{\cos 27^\circ} + \frac{1}{\sqrt{2}} \sin 27^\circ - \frac{1}{\sqrt{2}} \cancel{\cos 27^\circ} + \frac{1}{\sqrt{2}} \sin 27^\circ$$

$$= \frac{2}{\sqrt{2}} \sin 27^\circ$$

$$= \sqrt{2} \sin 27^\circ = \text{RHS.}$$

(i) Here,
 $\sin 35^\circ + \cos 35^\circ = \sqrt{2} \cos 10^\circ$

L.H.S.

$$\sin 35^\circ + \cos 35^\circ$$

$$= \sin(45^\circ - 10^\circ) + \cos(45^\circ - 10^\circ)$$

$$= (\sin 45^\circ \cos 10^\circ - \cos 45^\circ \sin 10^\circ) + (\cos 45^\circ \cos 10^\circ + \sin 45^\circ \sin 10^\circ)$$

$$= \frac{1}{\sqrt{2}} \cos 10^\circ - \frac{1}{\sqrt{2}} \sin 10^\circ + \frac{1}{\sqrt{2}} \cos 10^\circ + \frac{1}{\sqrt{2}} \sin 10^\circ$$

$$= \frac{2}{\sqrt{2}} \cos 10^\circ$$

$$= \sqrt{2} \cos 10^\circ = \text{R.H.S.}$$

Hence, proved.

(j) Here,

$$\sqrt{3} \cos 20^\circ + \sin 20^\circ = 2 \sin 80^\circ$$

L.H.S.

$$\sqrt{3} \cos 20^\circ + \sin 20^\circ$$

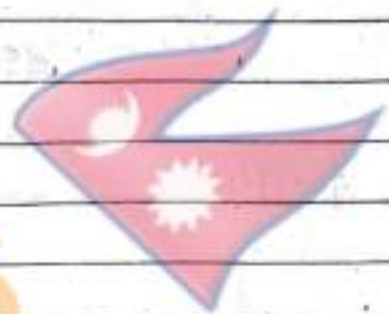
$$= 2 \left(\frac{\sqrt{3}}{2} \cos 20^\circ + \frac{1}{2} \sin 20^\circ \right)$$

$$= 2 (\sin 60^\circ \cos 20^\circ + \cos 60^\circ \sin 20^\circ)$$

$$= 2 \sin(60^\circ + 20^\circ)$$

$$= 2 \sin 80^\circ = \text{R.H.S.}$$

Hence, proved.



(K) Here,

$$\sin 65^\circ - \sin 25^\circ = \sqrt{2} \sin 20^\circ$$

L.H.S.

$$\sin 65^\circ - \sin 25^\circ$$

$$= \sin(45^\circ + 20^\circ) - \sin(45^\circ - 20^\circ)$$

$$= (\sin 45^\circ \cos 20^\circ + \cos 45^\circ \sin 20^\circ) - (\sin 45^\circ \cos 20^\circ - \cos 45^\circ \sin 20^\circ)$$

$$= \frac{1}{\sqrt{2}} \cos 20^\circ + \frac{1}{\sqrt{2}} \sin 20^\circ - \frac{1}{\sqrt{2}} \cos 20^\circ + \frac{1}{\sqrt{2}} \sin 20^\circ$$

$$= \frac{2}{\sqrt{2}} \sin 20^\circ$$

$$= \sqrt{2} \sin 20^\circ = \text{R.H.S.}$$

Hence, proved.

(1) Here;

$$\cos 20^\circ - \sin 20^\circ = \sqrt{2} \sin 25^\circ$$

L.H.S.

$$\cos 20^\circ - \sin 20^\circ$$

$$= \cos(45^\circ - 25^\circ) - \sin(45^\circ - 25^\circ)$$

$$= (\cos 45^\circ \cos 25^\circ + \sin 45^\circ \sin 25^\circ) - (\sin 45^\circ \cos 25^\circ - \cos 45^\circ \sin 25^\circ)$$

$$= \frac{1}{\sqrt{2}} \cos 25^\circ + \frac{1}{\sqrt{2}} \sin 25^\circ - \frac{1}{\sqrt{2}} \cos 25^\circ + \frac{1}{\sqrt{2}} \sin 25^\circ$$

$$= \frac{2}{\sqrt{2}} \sin 25^\circ$$

$$= \sqrt{2} \sin 25^\circ = \text{R.H.S.}$$

Hence, proved.

(m) Here,

$$\cos^2 15^\circ + \sin^2 15^\circ + \tan 15^\circ = 3 - \sqrt{3}$$

L.H.S.

$$\cos^2 15^\circ + \sin^2 15^\circ + \tan 15^\circ$$

$$= (\sin^2 15^\circ + \cos^2 15^\circ) + \tan(60^\circ - 45^\circ)$$

$$= 1 + \frac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \cdot \tan 45^\circ}$$

$$= 1 + \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{\sqrt{3} + 1 + \sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{2\sqrt{3}}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

$$= \frac{6 - 2\sqrt{3}}{3 - 1}$$

$$= \frac{6 - 2\sqrt{3}}{2}$$

$$= 3 - \sqrt{3} = \text{RHS.}$$

(n) Here,

$$\sin^2 15^\circ + \sin^2 45^\circ + \sin^2 75^\circ = \frac{3}{2}$$

L.H.S.

$$\sin^2 15^\circ + \sin^2 45^\circ + \sin^2 75^\circ$$

$$= \sin^2 15^\circ + \left(\frac{1}{\sqrt{2}}\right)^2 + \sin^2 (90^\circ - 15^\circ)$$

$$= \sin^2 15^\circ + \frac{1}{2} + \cos^2 15^\circ$$

$$= (\sin^2 15^\circ + \cos^2 15^\circ) + \frac{1}{2}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{2+1}{2}$$

$$= \frac{3}{2} = \text{R.H.S}$$

Hence, proved.

$$\{ \cot \theta \cdot \tan \theta = 1 \}$$

Q.15.(a) Here,

$$50^\circ - 40^\circ = 10^\circ$$

Taking $\tan$ on both sides,

$$\tan(50^\circ - 40^\circ) = \tan 10^\circ$$

$$\text{or, } \frac{\tan 50^\circ - \tan 40^\circ}{1 + \tan 50^\circ \cdot \tan 40^\circ} = \tan 10^\circ$$

$$\text{or, } \frac{\tan 50^\circ - \tan 40^\circ}{1 + \tan(90^\circ - 40^\circ) \cdot \tan 40^\circ} = \tan 10^\circ$$

$$\text{or, } \frac{\tan 50^\circ - \tan 40^\circ}{1 + \cot 40^\circ \cdot \tan 40^\circ} = \tan 10^\circ$$

$$\text{or, } \frac{\tan 50^\circ - \tan 45^\circ}{1 + 1} = \tan 10^\circ$$

$$\therefore \tan 50^\circ - \tan 45^\circ = 2 \tan 10^\circ$$

Hence, proved.

b) Here,
 $70^\circ - 20^\circ = 50^\circ$

$$\therefore \tan(70^\circ - 20^\circ) = \tan 50^\circ$$

$$\therefore \frac{\tan 70^\circ - \tan 20^\circ}{1 + \tan 70^\circ \tan 20^\circ} = \tan 50^\circ$$

$$\therefore \frac{\tan 70^\circ - \tan 20^\circ}{1 + \tan(90^\circ - 20^\circ) \tan 20^\circ} = \tan 50^\circ$$

$$\therefore \frac{\tan 70^\circ - \tan 20^\circ}{1 + \cot 20^\circ \tan 20^\circ} = \tan 50^\circ$$

$$\therefore \frac{\tan 70^\circ - \tan 20^\circ}{1 + 1} = \tan 50^\circ$$

$$\therefore \tan 70^\circ - \tan 20^\circ = 2 \tan 50^\circ$$

Hence, proved.

(c) Here,
 $80^\circ - 10^\circ = 70^\circ$

$$\text{or, } \tan(80^\circ - 10^\circ) = \tan 70^\circ$$

$$\text{or, } \frac{\tan 80^\circ - \tan 10^\circ}{1 + \tan 80^\circ \tan 10^\circ} = \tan 70^\circ$$

$$\text{or, } \frac{\tan 80^\circ - \tan 10^\circ}{1 + \tan(90^\circ - 10^\circ) \tan 10^\circ} = \tan 70^\circ$$

$$\text{or, } \frac{\tan 80^\circ - \tan 10^\circ}{1 + \cot 10^\circ \tan 10^\circ} = \tan 70^\circ$$

$$\text{or, } \frac{\tan 80^\circ - \tan 10^\circ}{1 + 1} = \tan 70^\circ$$

$$\text{or, } \frac{\tan 80^\circ - \tan 10^\circ}{2} = \tan 70^\circ$$

$$\therefore \tan 80^\circ - \tan 10^\circ = 2 \tan 70^\circ$$

Hence, proved.

(d) Here,

$$55^\circ - 35^\circ = 20^\circ$$

$$\tan(55^\circ - 35^\circ) = \tan 20^\circ$$

$$\text{or, } \frac{\tan 55^\circ - \tan 35^\circ}{1 + \tan 55^\circ \cdot \tan 35^\circ} = \tan 20^\circ$$

$$\text{or, } \frac{\tan 55^\circ - \tan 35^\circ}{1 + \tan(90^\circ - 35^\circ) \cdot \tan 35^\circ} = \tan 20^\circ$$

$$\text{or, } \frac{\tan 55^\circ - \tan 35^\circ}{1 + \cot 35^\circ \cdot \tan 35^\circ} = \tan 20^\circ$$

$$\text{or, } \frac{\tan 55^\circ - \tan 35^\circ}{1 + 1} = \tan 20^\circ$$

$$\text{or, } \frac{\tan 55^\circ - \tan 35^\circ}{2} = \tan 20^\circ$$

$$\therefore \tan 55^\circ - \tan 35^\circ = 2 \tan 20^\circ$$

Hence, proved.

(e) Here,

$$65^\circ - 25^\circ = 40^\circ$$

$$\text{or, } \tan(65^\circ - 25^\circ) = \tan 40^\circ$$

$$\text{or, } \frac{\tan 65^\circ - \tan 25^\circ}{1 + \tan 65^\circ \tan 25^\circ} = \tan 40^\circ$$

$$\text{or, } \frac{\tan 65^\circ - \tan 25^\circ}{1 + \tan(90^\circ - 25^\circ) \tan 25^\circ} = \tan 40^\circ$$

$$\text{or, } \frac{\tan 65^\circ - \tan 25^\circ}{1 + \cot 25^\circ \cdot \tan 25^\circ} = \tan 40^\circ$$

$$\text{or, } \frac{\tan 65^\circ - \tan 25^\circ}{1 + 1} = \tan 40^\circ$$

$$\text{or, } \frac{\tan 65^\circ - \tan 25^\circ}{2} = \tan 40^\circ$$

$$\therefore \tan 65^\circ - \tan 25^\circ = 2 \tan 40^\circ$$

Hence, proved.

Q. 16(a) Here,

$$22^\circ + 23^\circ = 45^\circ$$

$$\tan(22^\circ + 23^\circ) = \tan 45^\circ$$

$$\text{or, } \frac{\tan 22^\circ + \tan 23^\circ}{1 - \tan 22^\circ \tan 23^\circ} = 1$$

$$\text{or, } \tan 22^\circ + \tan 23^\circ = 1 - \tan 22^\circ \tan 23^\circ$$

$$\therefore \tan 23^\circ + \tan 22^\circ + \tan 22^\circ \tan 23^\circ = 1$$

Hence, proved.

(b) Here,

$$20^\circ + 25^\circ = 45^\circ$$

$$\tan(20^\circ + 25^\circ) = \tan 45^\circ$$

$$\text{or, } \frac{\tan 20^\circ + \tan 25^\circ}{1 - \tan 20^\circ \tan 25^\circ} = 1$$

$$\therefore 1 - \tan 20^\circ \tan 25^\circ = \tan 20^\circ + \tan 25^\circ$$

Hence, proved

(c) Here,

$$32^\circ + 13^\circ = 45^\circ$$

$$\tan(32^\circ + 13^\circ) = \tan 45^\circ$$

$$\therefore \frac{\tan 32^\circ + \tan 13^\circ}{1 - \tan 32^\circ \tan 13^\circ} = 1$$

$$\therefore 1 - \tan 32^\circ \tan 13^\circ = \tan 32^\circ + \tan 13^\circ$$

Hence, proved.

Q. 17 Solⁿ.

(a) Here,

$$28^\circ + 72^\circ + 80^\circ = 180^\circ$$

$$\tan(28^\circ + 72^\circ + 80^\circ) = \tan 180^\circ$$

$$\therefore \frac{\tan 28^\circ + \tan 72^\circ + \tan 80^\circ - \tan 28^\circ \tan 72^\circ \tan 80^\circ}{1 - \tan 28^\circ \tan 72^\circ - \tan 72^\circ \tan 80^\circ - \tan 80^\circ \tan 28^\circ} = 0$$

$$\therefore \tan 28^\circ + \tan 72^\circ + \tan 80^\circ - \tan 28^\circ \tan 72^\circ \tan 80^\circ = 0$$

$$\therefore \tan 28^\circ + \tan 72^\circ + \tan 80^\circ = \tan 28^\circ \tan 72^\circ \tan 80^\circ$$

Hence, proved.

(b) Here,
 $50^\circ + 60^\circ + 70^\circ = 180^\circ$

$$\tan(50^\circ + 60^\circ + 70^\circ) = \tan 180^\circ$$

$$\text{or, } \frac{\tan 50^\circ + \tan 60^\circ + \tan 70^\circ - \tan 50^\circ \tan 60^\circ \tan 70^\circ}{1 - \tan 50^\circ \tan 60^\circ - \tan 60^\circ \tan 70^\circ - \tan 70^\circ \tan 50^\circ} = 0$$

$$\text{or, } \tan 50^\circ + \tan 60^\circ + \tan 70^\circ - \tan 50^\circ \tan 60^\circ \tan 70^\circ = 0$$

$$\therefore \tan 50^\circ + \tan 60^\circ + \tan 70^\circ = \tan 50^\circ \tan 60^\circ \tan 70^\circ$$

Hence, proved.

Q.18 (a) Here,
 $3A + 4A = 7A$

$$\tan(3A + 4A) = \tan 7A$$

$$\text{or, } \frac{\tan 3A + \tan 4A}{1 - \tan 3A \tan 4A} = \tan 7A$$

$$\text{or, } \tan 3A + \tan 4A = \tan 7A - \tan 7A \cdot \tan 3A \tan 4A$$

$$\therefore \tan 7A - \tan 4A - \tan 3A = \tan 7A \tan 3A \tan 4A$$

Hence, proved.

(b) Here,
 $9A + 4A = 13A$

$$\tan(9A + 4A) = \tan 13A$$

$$\therefore \frac{\tan 9A + \tan 4A}{1 - \tan 9A \cdot \tan 4A} = \tan 13A$$

$$\therefore \tan 9A + \tan 4A = \tan 13A - \tan 9A \cdot \tan 4A \cdot \tan 13A$$

$$\therefore \tan 13A = \tan 4A + \tan 9A + \tan 4A \tan 9A \tan 13A$$

Hence, proved.

Q.19 Solⁿ.

(a) Here,
 $\tan x = \frac{2}{3}$

$$\tan y = \frac{1}{5}$$

Now,

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y}$$

$$\therefore \tan(x+y) = \frac{\frac{2}{3} + \frac{1}{5}}{1 - \frac{2}{3} \times \frac{1}{5}}$$

$$\therefore \tan(X+Y) = \frac{2 \times 5 + 1 \times 3}{1 - \frac{2}{15}}$$

$$\therefore \tan(X+Y) = \frac{10+3}{15-2}$$

$$\therefore \tan(X+Y) = \frac{13}{13}$$

$$\therefore \tan(X+Y) = 1$$

$$\therefore \tan(X+Y) = \tan 45^\circ$$

$$\therefore (X+Y) = 45^\circ$$

Hence, proved.

(b) Here,
 $\tan A = \frac{1}{2}$

$\tan B = \frac{1}{3}$

Now,
 $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\therefore \tan(A+B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}$

$\therefore \tan(A+B) = \frac{3+2}{6} \div \frac{6-1}{6}$

$\therefore \tan(A+B) = \frac{5}{5}$

$\therefore \tan(A+B) = 1$

$\therefore \tan(A+B) = \tan\left(\frac{\pi}{4}\right)$

$\therefore A+B = \frac{\pi}{4}$

Hence, proved

(c) Here,
 $\tan A = \frac{3}{4}$

$\tan B = \frac{1}{7}$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore \tan(A+B) = \frac{\frac{3}{4} + \frac{1}{7}}{1 - \frac{3}{4} \times \frac{1}{7}}$$

$$\therefore \tan(A+B) = \frac{\frac{21+4}{28}}{\frac{28-3}{28}}$$

$$\therefore \tan(A+B) = \frac{25}{25}$$

$$\therefore \tan(A+B) = 1$$

$$\therefore \tan(A+B) = \tan\left(\frac{\pi}{4}\right)$$

$$\therefore A+B = \frac{\pi}{4}$$

Hence, proved.

(d) Here,
 $\tan A = m$
 $\tan B = \frac{1}{m}$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\therefore \tan(A+B) = \frac{m + \frac{1}{m}}{1 - m \times \frac{1}{m}}$$

$$\therefore \tan(A+B) = \frac{\frac{m^2 + 1}{m}}{1 - 1}$$

$$\therefore \tan(A+B) = \frac{m^2 + 1}{0 \times m}$$

$$\therefore \tan(A+B) = \frac{m^2 + 1}{0}$$

$$\therefore \tan(A+B) = \infty$$

$$\therefore \tan(A+B) = \tan 90^\circ$$

$$\therefore A+B = 90^\circ$$

Hence, proved.

Q. 20. 80/11.

(a) Here,
 $A+B = 45^\circ$

$$\text{or, } \tan(A+B) = \tan 45^\circ$$

$$\text{or, } \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$$

$$\text{or, } \tan A + \tan B = 1 - \tan A \cdot \tan B$$

$$\therefore \tan A + \tan B + \tan A \cdot \tan B = 1$$

(b) Here,
 $A+B = 45^\circ$

$$\tan(A+B) = \tan 45^\circ$$

$$\text{or, } \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = 1$$

$$\text{or, } \tan A + \tan B = 1 - \tan A \cdot \tan B$$

$$\text{or, } \tan A + \tan B + \tan A \cdot \tan B = 1$$

$$\text{or, } 1 + \tan A + \tan B + \tan A \tan B = 1 + 1$$

$$\text{or, } 1(1 + \tan A) + \tan B(1 + \tan A) = 2$$

$$\therefore (1 + \tan A)(1 + \tan B) = 2$$

(c) Here,
 $A + B = 45^\circ$

$$\cot(A + B) = \cot 45^\circ$$

$$\frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$$

$$\text{or, } \cot A \cot B - 1 = \cot B + \cot A$$

$$\text{or, } \cot A \cot B - \cot B - \cot A = 1$$

$$\text{or, } \cot A \cot B - \cot B - \cot A + 1 = 1 + 1$$

$$\text{or, } \cot B(\cot A - 1) - 1(\cot A - 1) = 2$$

$$\text{or, } (\cot A - 1)(\cot B - 1) = 2$$

Hence, proved.

(d) Here,
 $A+B=45^\circ$

$$\cot(A+B) = \cot 45^\circ$$

$$\text{or, } \frac{\cot A \cot B - 1}{\cot B + \cot A} = 1$$

$$\text{or, } \cot A \cot B - 1 = \cot B + \cot A$$

$$\text{or, } \cot A \cot B = \cot B + \cot A + 1 \rightarrow (1)$$

L.H.S.

$$(1 + \cot A)(1 + \cot B)$$

$$= 1 + \cot B + \cot A + \cot A \cot B$$

$$= (1 + \cot B + \cot A) + \cot A \cot B \text{ [from eqn. (1)]}$$

$$= \cot A \cot B + \cot A \cot B$$

$$= 2 \cot A \cot B = \text{RHS.}$$

Hence, proved.

Q.21 Solⁿ

(a) Here,

$$A+B = 225^\circ$$

$$\text{or, } \tan(A+B) = \tan 225^\circ$$

$$\text{or, } \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\therefore \tan A + \tan B = 1 - \tan A \cdot \tan B$$

Hence, proved.

(b) Here,

$$A-B = 45^\circ$$

$$\text{or, } \tan(A-B) = \tan 45^\circ$$

$$\text{or, } \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B} = 1$$

$$\text{or, } \tan A - \tan B = 1 + \tan A \cdot \tan B$$

$$\tan A = 1 + \tan B + \tan A \cdot \tan B \rightarrow (1)$$

L.H.S.

$$(1 + \tan A)(1 + \tan B)$$

$$= 1 + \tan B + \tan A + \tan A \tan B$$

$$= (1 + \tan B + \tan A \tan B) + \tan A$$

$$= \tan A + \tan A \quad [\text{From eqn. (1)}]$$

$$= 2 \tan A = \text{RHS.}$$

Hence, proved.

Q.22(a) Here,

$$\frac{\sin(A-B)}{\cos A \cos B} = \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}$$

$$= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B}$$

$$= \tan A - \tan B$$

Similarly,

$$\frac{\sin(B-C)}{\cos B \cos C} = \tan B - \tan C$$

$$\frac{\sin(C-A)}{\cos A \cos C} = \tan C - \tan A$$

Now,

L.H.S.

$$\frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos A \cos C}$$

$$= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A$$

$$= 0 = \text{RHS}$$

Hence, proved.

(b) Here,

$$\frac{\sin(A-B)}{\sin A \sin B} = \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B}$$

$$= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B}$$

$$= \cot B - \cot A$$

Similarly,

$$\frac{\sin(B-C)}{\sin B \cdot \sin C} = \cot C - \cot B$$

$$\frac{\sin(C-A)}{\sin A \cdot \sin C} = \cot A - \cot C$$

Now,

L.H.S.

$$\frac{\sin(A-B)}{\sin A \cdot \sin B} + \frac{\sin(B-C)}{\sin B \cdot \sin C} + \frac{\sin(C-A)}{\sin A \cdot \sin C}$$

$$= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C$$

$$= 0 = \text{RHS}$$

Hence, proved.

(c) Here,

L.H.S.

$$\sin A \cdot \sin(B-C) + \sin B \cdot \sin(C-A) + \sin C \cdot \sin(A-B)$$

$$= \sin A (\sin B \cos C - \cos B \sin C) + \sin B (\sin C \cos A - \cos C \sin A) + \sin C (\sin A \cos B - \cos A \sin B)$$

$$= \sin A \sin B \cos C - \sin A \cos B \sin C + \cos A \sin B \sin C - \sin A \sin B \cos C + \sin A \cos B \sin C - \cos A \sin B \sin C$$

$$= 0 = \text{RHS}$$

(c) Here,

$$\sin(A+B+C) = \sin(A+B)\cos C + \cos(A+B)\sin C$$

$$= (\sin A \cos B + \cos A \sin B) \cos C + (\cos A \cos B - \sin A \sin B) \sin C$$

$$= \sin A \cos B \cdot \cos C + \cos A \sin B \cos C + \cos A \cos B \cdot \sin C - \sin A \sin B \sin C$$

$$= \cos A \cos B \cdot \cos C \left(\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} + \frac{\sin C}{\cos C} - \right.$$

$$\left. \frac{\sin A \sin B \sin C}{\cos A \cos B \cos C} \right)$$

$$= \cos A \cos B \cos C (\tan A + \tan B + \tan C - \tan A \tan B \tan C)$$

$$= \text{R.H.S.}$$

Hence, proved

(e) Here,
 $\cos(A+B+C)$

$$= \cos(A+B)\cos C - \sin(A+B)\sin C$$

$$= (\cos A \cos B - \sin A \sin B)\cos C - (\sin A \cos B + \cos A \sin B)\sin C$$

$$= \cos A \cos B \cos C - \sin A \sin B \cos C - \sin A \cos B \sin C - \cos A \sin B \sin C$$

$$= \cos A \cos B \cos C \left(1 - \frac{\sin A \sin B}{\cos A \cos B} - \frac{\sin A \sin C}{\cos A \cos C} - \frac{\sin B \sin C}{\cos B \cos C} \right)$$

$$= \cos A \cos B \cos C (1 - \tan A \tan B - \tan A \tan C - \tan B \tan C)$$

$$= \cos A \cos B \cos C (1 - \tan B \tan C - \tan C \tan A - \tan A \tan B)$$

$$= \text{RHS.}$$

Hence, proved.

(f) Here,
 $\tan(A+B+C)$

$$= \frac{\sin(A+B+C)}{\cos(A+B+C)}$$

$$= \frac{\cos A \cos B \cos C (\tan A + \tan B + \tan C - \tan A \tan B \tan C)}{\cos A \cos B \cos C (1 - \tan B \tan C - \tan C \tan A - \tan A \tan B)}$$

$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan B \tan C - \tan C \tan A - \tan A \tan B}$$

$$= \text{R.H.S.}$$

Hence, proved